PERSPECTIVE:

OR, THE

ART of drawing the Repreſentations of all OBJECTS upon a Plane.

In TWO SECTIONS.

SECT. I. Demonſtrates the Principles whereon this Art is founded.

SECT. II. Gives the practical RULES for Operation; with great Variety of Examples.

Decipimur ſpecie recti: ——— HOR.

PREFACE

THE

PREFACE.

THE Art of Perſpective *is properly a branch of Optics; for it depends on this optic principle, that all the rays of light flowing from the ſeveral parts of an object, move in right lines to the eye. And therefore the images of theſe parts muſt be ſomewhere in theſe viſual rays; and conſequently muſt be where theſe lines interſect the picture. Perſpective is a kind of projection, for here the picture being placed at a convenient diſtance between the eye and the object, (or it may as eaſily be placed beyond the object;) the eye projects the ſeveral points, and parts, of the body, upon that picture, by lines drawn from the eye to the parts of the body; and that by certain rules, which are to be explained in this book.*

Perſpective is an art abſolutely neceſſary to be underſtood by all thoſe that practice drawing. From this it derives all its beauty and juſtneſs. It is alſo of great uſe in Architecture, Fortification, Engraving, Carving, and all the mechanical arts; and particularly in Painting. It is of admirable uſe for deſcribing the figures of ſolids, or of any buildings, and of all ſorts of machines. An artiſt cannot put his deſign into execution without this; for all neat and curious drawing ſignifies nothing without it, if the deſign is not executed according to the rules of this art. By this, the figures and ſhapes of all natural objects may be imitated to the life, which no other art can equal. By

this are contrived those deceptions in vision, which so agreeably delight and please the eye.

I took notice in the Optics *how delightful a study that is; but the greatest share of that pleasure belongs to Perspective. For what greater pleasure can one enjoy by the eye, than to see a grand building, a large country, or great city, so truly drawn and finely coloured, which being placed at a due distance, shall appear to the eye in its true proportions, and in all its pomp and beauty, even to vie with Nature itself; and even to out-do it in some cases. For if it be drawn in summer, with all the beauty of flowers, and verdure of leaves; such a draught look'd at in winter will appear gay; whilst the place itself, now destitute of all these beauties and natural colourings, gives but a disagreeable prospect? Can it fail to please and charm the beholder to see a whole country have the very same appearance, and the same beauty, as the place it was drawn for, and even of the same magnitude when seen thro' a lens at a due distance. And if it be a place one knows, how agreeably one is surprized to find himself in it. Who would not be struck at so agreeable a delusion?*

Hence it has been objected by some, that this art does not shew us truth, but deceives us with false appearances; alledging, that nothing appears to us as it really is, but in a quite different form. It is true, if we regard the picture itself as an object, which is a piece of paper, being nothing but a plane surface perpendicular to the horizon, with several lines and colourings laid on it; then it gives us a false appearance of itself. But nobody that draws a picture, ever intends that it should represent itself, and therefore this objection will be nothing to the purpose. For the picture is always designed to represent a thing that is out of the picture; and therefore if the picture truly represents that thing, then it shews us truth. And if it does not in all respects, represent it truly; then so far

as

as that fails, it does not repreſent the thing intended, but ſomething elſe; and then indeed it ſhews us falſhood. So that if the picture is truly drawn, it ſhews truth; but it never ſhews falſhood, but when it is falſely drawn.

As the art of painting has always been in great eſteem, therefore we need not wonder that men of excellent talents, have taken great pains in perfecting that art. But without the knowledge of Perſpective, that art will always be deficient and imperfect; and therefore every painter that is ignorant of Perſpective will unavoidably run into errors. Perſpective then is the compleating of the art of painting, without which no painting can be perfect. And therefore no perſon ſhould engage in the art of painting, except he reſolve to make himſelf acquainted with the rules of Perſpective.

The inventive part of painting, is nothing but the original deſign, contrived in the imagination of the artiſt; but the executive part is wholly tied to the rules of art, which muſt not be tranſgreſſed upon any account. Therefore he that deſigns to be a painter, muſt firſt of all learn Perſpective; and when he has done this, and informed his judgment what alterations happen to figures, when drawn upon a plane; he ought then to be put to drawing by ſight, and exerciſe himſelf in this along with Perſpective, till he becomes ſufficiently perfect in both. Nothing ought to be more familiar to a painter than Perſpective; for it is the only thing that can make the judgment correct; and will help the fancy to invent with ten times more eaſe, than it could do without it.

Yet it is not neceſſary or even poſſible, that the ſtrict rules of Perſpective ſhould be applied to the drawing every minute part of a body. For there would be ſuch a confuſion of lines, and their interſections would be ſo near together, that no exactneſs could be expected from thence; ſuch parts as theſe ought to be drawn by the

eye.

eye. Nevertheleſs, the rules of Perſpective muſt direct ones judgment, and be a guide to his eye, how they are to be drawn.

In the firſt ſection of this book, I have laid down the fundamental propoſitions on which the practical part of Perſpective is founded; and given their demonſtrations, that their truth and certainty may appear. And in the ſecond ſection, I have given all the rules of practice, for performing the ſeveral operations, in drawing any points, lines or ſolids in Perſpective. And have given the reaſons and demonſtrations of theſe rules, exceeding ſhort; being eaſily deduced from the propoſitions in the firſt ſection. And have alſo given great plenty of examples to illuſtrate the rules, and to ſatisfy ſuch as have a great deſire for obtaining the arts of drawing and painting. And by frequent uſe and practice ſeveral compendious methods of working will occur of themſelves to every practitioner. All this is principally for upright pictures, the only ſort that are uſeful. For as to inclined pictures, they require more lines and more labour; and no object drawn in them ever appears ſo well as in an upright picture; ſo that they are nothing but a piece of curioſity. Yet leaſt it ſhould happen that any object requires to be drawn upon an oblique plane, which cannot be put into any other poſition; I have alſo ſhewn the properties of theſe pictures, and the methods of drawing upon them, in a general way; with ſeveral examples thereto, which will make the method very plain and clear, to any body that is deſirous to know how to draw upon this kind of pictures.

W. Emerſon.

Fig.

PERSPECTIVE.

DEFINITIONS.

DEF. I.

PERSPECTIVE is the art of repreſenting any object upon a plane; juſt as it appears to the eye, in any given ſituation. When the plane is perpendicular to the horizon, it is called *common Perſpective.* When the eye is infinitely diſtant from that plane, it is called *military Perſpective.*

DEF. II.

The *Picture* is that plane upon which the object is repreſented, as ABCD; which is perpendicular to the horizon; and is ſuppoſed as tranſparent as glaſs. This is alſo called the *Table*, or *perſpective Plane.* 1.

DEF. III.

The *geometrical Plane* is a plane parallel to the horizon, in which the viſible objects are ſituated as FGHI. And the *ſituation* of any point of an object, is where a perpendicular from that point, cuts this plane.

DEF. IV.

The *horizontal Plane* is a plane paſſing thro' the eye, parallel to the horizon, as KLMN; or rather in the horizon. Therefore this is parallel to the geometrical plane, and perpendicular to the picture.

DEF. V.

The *vertical Plane* is a plane paſſing thro' the eye, perpendicular to the picture, and to the horizon, as OPQR.

eye. Nevertheleſs, the rules of Perſpective muſt direct ones judgment, and be a guide to his eye, how they are to be drawn.

In the firſt ſection of this book, I have laid down the fundamental propoſitions on which the practical part of Perſpective is founded; and given their demonſtrations, that their truth and certainty may appear. And in the ſecond ſection, I have given all the rules of practice, for performing the ſeveral operations, in drawing any points, lines or ſolids in Perſpective. And have given the reaſons and demonſtrations of theſe rules, exceeding ſhort; being eaſily deduced from the propoſitions in the firſt ſection. And have alſo given great plenty of examples to illuſtrate the rules, and to ſatisfy ſuch as have a great deſire for obtaining the arts of drawing and painting. And by frequent uſe and practice ſeveral compendious methods of working will occur of themſelves to every practitioner. All this is principally for upright pictures, the only ſort that are uſeful. For as to inclined pictures, they require more lines and more labour; and no object drawn in them ever appears ſo well as in an upright picture; ſo that they are nothing but a piece of curioſity. Yet leaſt it ſhould happen that any object requires to be drawn upon an oblique plane, which cannot be put into any other poſition; I have alſo ſhewn the properties of theſe pictures, and the methods of drawing upon them, in a general way; with ſeveral examples thereto, which will make the method very plain and clear, to any body that is deſirous to know how to draw upon this kind of pictures.

W. Emerſon.

Fig.

PERSPECTIVE.

DEFINITIONS.

DEF. I.

PERSPECTIVE is the art of repreſenting any object upon a plane; juſt as it appears to the eye, in any given ſituation. When the plane is perpendicular to the horizon, it is called *common Perſpective.* When the eye is infinitely diſtant from that plane, it is called *military Perſpective.*

DEF. II.

The *Picture* is that plane upon which the object is repreſented, as ABCD; which is perpendicular to the horizon; and is ſuppoſed as tranſparent as glaſs. This is alſo called the *Table*, or *perſpective Plane.* 1.

DEF. III.

The *geometrical Plane* is a plane parallel to the horizon, in which the viſible objects are ſituated as FGHI. And the *ſituation* of any point of an object, is where a perpendicular from that point, cuts this plane.

DEF. IV.

The *horizontal Plane* is a plane paſſing thro' the eye, parallel to the horizon, as KLMN; or rather in the horizon. Therefore this is parallel to the geometrical plane, and perpendicular to the picture.

DEF. V.

The *vertical Plane* is a plane paſſing thro' the eye, perpendicular to the picture, and to the horizon, as OPQR.

Fig. 1.

DEF. VI.

The *directing Plane* is a plane paſſing thro' the eye, parallel to the picture, as KNHI. And its interſection IH with the geometrical plane, is the *Line of Extremes.*

DEF. VII.

The *ground Line*, is the ſection of the picture, and the geometrical plane, as DC. It is alſo called the *Baſe Line*, or *fundamental Line.*

DEF. VIII.

The *horizontal Line* is the interſection of the picture, and the horizontal plane, as TV. Whence this is parallel to the ground line. It is alſo called the *vaniſhing Line.* But in a general ſenſe, the *vaniſhing Line* of any plane, is the ſection of its parallel plane drawn thro' the eye, with the picture.

DEF. IX.

The *principal Ray*, is a line drawn from the eye, perpendicular to the picture, as ES. Therefore this lies in the horizontal plane.

DEF. X.

The *Line of Station*, is the ſection of the vertical plane with the geometrical plane, as RQ.

DEF. XI.

The *Line of Diſtance* is that part of the line of ſtation, contained between the directing plane, and the picture; as RX. Hence the line of diſtance is parallel and equal to the principal ray.

DEF. XII.

The *vertical Line* is the line in which the vertical plane cuts the picture; as SX.

DEF. XIII.

The *Hight of the Eye* is a perpendicular paſſing from the eye to the geometrical plane, as ER. This is equal and parallel to the vertical line.

DEF.

DEF. XIV. Fig. 1.

The *Point of View* is the point where the eye is fixed, as E.

DEF. XV.

The *Point of Sight* is the point where the principal ray cuts the picture, as S. Therefore this is in the horizontal and vertical lines. It is alſo called the *principal Point*, or *Center of the Picture*.

DEF. XVI.

The *Point of Diſtance* is a point in the horizontal line, whoſe diſtance from the point of ſight is equal to the principal ray, as V or T; ſuppoſing VS or TS, equal to ES.

DEF. XVII.

The *accidental Point* of a right line is the point, where a line, drawn thro' the eye, parallel to the line propoſed, cuts the picture. This is alſo called the *vaniſhing Point*. Hence all lines parallel to one another, have the ſame accidental point. And all lines perp. to the picture, have the point of ſight for their accidental point. And all lines parallel to the picture have no accidental point.

DEF. XVIII.

The *Plan* is the projection of an object upon the geometrical plane, by lines drawn from all points of the object, perpendicular to that plane. This is alſo called the *Ichnography* of the object, or the *Baſe* of it.

DEF. XIX.

The *Front* is the fore view of an object, or the ſection of it parallel to the picture; or it is the projection of it upon a plane parallel to the picture.

DEF. XX.

The *Profil* is the ſide view, or ſection of an object, parallel to the vertical plane. It is taken for the appearance of any ſide.

DEF. XXI.

Fig. 1. *Schenography* is the repreſenting ſolids, or ſuch objects as are elevated in the air.

DEF. XXII.

The *optic Cone* is the cone of rays proceeding from the eye to all the points of an object, and cutting the picture.

SCHOLIUM.

In general, the things to be drawn, are called *Objects* or *Originals*; and their pictures when drawn, are called *Repreſentations*, *Appearances*, *Figures*, *Images*, or *Projections*.

AXIOMS.

1.

The appearanee or repreſentation of any point of an object, is where the right line, drawn from the eye to that point of the object, cuts the picture.

2.

If any part of an object touches the picture, its appearance will be in that part of the picture which it touches.

3.

All parts of objects lower than the eye, or below the horizontal plane, will be repreſented below the horizontal line, in the picture: and thoſe that are higher, will be repreſented above it.

4.

All objects on the right hand of the vertical plane, will be repreſented on the right hand of the vertical line; and thoſe on the left hand, on the left ſide of it.

5.

If a right line in the object paſſes thro' the eye, it will be repreſented in the picture by a ſingle point.

6.

Any plane (in the object) paſſing thro' the eye, will appear only a right line in the picture.

SECT.

Fig.

SECT. I.

The fundamental Principles of PERSPECTIVE.

PROP. I.

IN military perſpective, lines that are parallel in the object, are repreſented by parallel lines in the picture. And they are ſhortened in the picture, in the ratio of the radius to the ſine of inclination of the viſual rays.

For let there be two parallel lines in any object; and ſuppoſe two planes to paſs thro' theſe two lines and the eye. Now ſince the eye is ſuppoſed infinitely diſtant, in this ſort of perſpective; therefore theſe planes, which only interſect at an infinite diſtance, may be looked upon as parallel. But (Geom. V. 11.) if two parallel planes be cut by a third; their common ſections are parallel. And therefore the two planes cut the picture in two parallel lines; which repreſent the two parallel lines in the object. And it is equaliy the ſame if there is never ſo many of theſe parallel lines. And it is evident that their lengths are diminiſhed in the ratio of radius to the ſine of obliquity.

Cor. 1. *If the eye be at a great diſtance; lines that are parallel in the object, are repreſented by lines nearly parallel in the picture.*

Cor. 2. *Viſual rays from the eye, cut all parallels, and all parts of them, at the ſame angle.*

This follows from the paralleliſm, both of the rays and the lines.

Cor. 3. *Equal parallel lines have their images equal.*

Fig.

PROP. II.

2. *In common perſpective, a right line in the object, which paſſes not thro' the eye, is repreſented by a right line in the picture.*

Let GH be the right line, E the eye; draw EG, EH, cutting the picture in *g* and *h*; then *g* and *h*, repreſent the points G and H (by Ax. 1.). And thus all the points between G and H are repreſented by right lines drawn from thence to E; and all theſe lines are in the plane EGH. And ſince that plane interſects the plane AC in the right line *gh* (Geom. B. V. Prop. 3.) therefore all the points in GH are repreſented by all the points in *gh*; or the right line GH by the right line *gh*.

Cor. *Hence the lines* GH *and gh, are in one and the ſame plane.*

PROP. III.

If a right line be parallel to the picture, its repreſentation in the picture, will be parallel to that right line.

2. For let GH be a line parallel to the picture, and ſuppoſe a plane to paſs thro' it parallel to the picture, then the plane EGH being cut by the two parallel planes GH, and AC; the ſections GH, *gh* will be parallel (by Prop. XI. B. V. Geometry.)

Cor. 1. *If a right line be parallel to the ground line; its appearance in the picture will be parallel to the ground line.*

Cor. 2. *If a right line be parallel to the vertical line; its appearance in the picture will be perpendicular to the ground line.*

Cor.

Cor. 3. *If a line parallel to the picture interſect the geometrical plane, and make any angle with it; its figure or appearance in the picture will make the ſame angle with the ground line.* Fig. 2.

Cor. 4. *If two lines parallel to the picture, interſect; their repreſentations in the picture, will make the ſame angle, as the original lines.*

For each of them makes the ſame angle as its original; and therefore the differences of each two are equal; that is, the angles which the originals and their repreſentations make, are equal.

Cor. 5. *All theſe things hold good, when the picture is inclined to the horizon.*

PROP. IV.

The figure or appearance of any right line in the picture, tends to the accidental point of that line.

Let GH be the original line, *gh* its figure or appearance in the picture, P the accidental point of it, E the eye. Then (Def. 17.) ſince EP and HG are parallel; EP and HG are in the ſame plane, which plane paſſes thro' E and P. Likewiſe (Cor. Prop. II.) *gh* is in the plane EGH; therefore all the points P, *g*, *h* are in one plane, which plane interſects the picture in the line P*gh*; therefore *gh* produced, paſſes thro' P. 3.

Cor. 1. *The repreſentations of all lines parallel to* GH, *do all meet in the accidental point* P.

Cor. 2. *The appearances of all lines perpendicular to the picture, tend to the point of ſight.*

Cor. 3. *The appearances of all lines, in or parallel to, the geometrical plane, making an angle of* 45 *degrees with the ground line or picture; tend to the point of diſtance on that ſide.*

Fig. 3. Cor. 4. *The appearance of a right line, paſſes both thro' the accidental point, and thro' the point where the original cuts the picture.*

PROP. V.

If two equal right lines be parallel to, and equidiſtant from, the picture; their repreſentations will be equal.

4. Let the lines GH, KI, be equal, and both parallel to the picture AC, and at equal diſtances from it; and *gh*, *ki* their repreſentations. Draw HI and *hi*. E is the eye. Then by reaſon of the parallel planes GHIK and AC, the triangles EGH, *egh*; alſo EHI, *ehi*; and EIK, E*ik*, are ſimilar each to each. Therefore, GH : *gh* : : EH : E*h* :: HI : *hi* : : EI : E*i* : : IK : *ik*. But GH = IK, therefore *gh* = *ik*.

Cor. 1. *If ſeveral lines are parallel to, and equidiſtant from, the picture; their repreſentations will be to one another, in the ſame proportion as the original lines.*

For we have GH : *gh* : : IK : *ik*; or GH : IK :: *gh* : *ik*.

Cor. 2. *If a right line parallel to the picture, be divided into equal parts; the images or repreſentations of theſe ſeveral parts, will be equal.*

Cor. 3. *Two equal right lines proceeding from one point, and both parallel to the picture; will have their appearances equal in the picture. And the angle they make in the picture, will be equal to the angle made by the originals.*

For the repreſentations are parallel to the originals, by Prop. III.

Cor. 4. *All theſe things hold equally for an inclined picture.*

PROP.

Fig.

PROP. VI.

If a circle be parallel to the picture, its figure in the picture will be a circle, in any ſort of projection.

Let E be the eye, BD the circle. Then ſince the optic cone EBD, is cut by the plane of the picture AC, parallel to the baſe of the cone, therefore (by Cor. Prop. LXXXIX. B. I. Conic Sections) the ſection *bd* will be a circle. 5.

Cor. 1. *If a circle be ſo placed, that the optic cone, be cut by the picture, ſo that the angles on oppoſite ſides may be equal, which are made by the picture and by the baſe; then alſo the appearance in the picture will be a circle.*

For then it is cut in ſub-contrary poſition; and therefore (by the ſame Prop.) the ſection is a circle.

Cor. 2. *In other caſes when a circle is not parallel to the picture; its appearance is an ellipſis; ſuppoſing the optic cone to be cut on both ſides by the picture.*

PROP. VII.

The appearance of any plane figure parallel to the picture, will be ſimilar to the original. 5.

Let GHIK be the plain figure whoſe repreſentation is *ghik*. Then ſince the optic pyramid EGHIK is cut by the plane of the picture AC, parallel to the baſe GHIK; therefore the ſection *ghik* will be ſimilar to GHIK (by Cor. 1. Prop. XVII. B. VI. Geometry.)

PROP. VIII.

The vaniſhing or accidental points, of all lines drawn in any plane, are in the vaniſhing line of that plane. 6.

Let FG be any line in the plane CFG, and EL drawn from the eye at E, parallel to FG, cutting the

Fig. 6. the picture in L. Then ſince all lines as FG are in the plane CFG, all the parallels EL, are in the parallel plane AEL; and therefore all the points L, are in the interſection AL; that is, in the vaniſhing line. And it is the ſame thing whether the plane AC be right or oblique to the plane CG, ſince the planes CFG and EAL are parallel in all caſes.

PROP. IX.

7. *All lines perpendicular to the picture, will be projected into ſome parts or other of the ſame lines in the picture; in whatever point of the principal ray, the eye is placed.*

Let FG be a line perpendicular to the picture, E the eye, ES the principal ray; produce GF to cut the picture in N, and draw SN. Then ſince ES, NG are parallel, they and the line SN are in one plane. Therefore FG appears ſomewhere in the line SN. Then let the eye remove to K, in the ſame line ES, and K is ſtill in the ſame plane ESNG; therefore FG appears ſtill in the line SN, which has not changed its ſituation. But then the repreſentation of FG does not fall in the ſame part of the line SN, when the eye is at E, as when it is at K; for the farther the point K recedes back, the leſs angle the line FG appears under; and the further the repreſentation of it is from S.

Cor. 1. *The further the eye recedes along the principal ray, the leſs any line appears, which is perpendicular to the picture.*

Cor. 2. *Wherever the eye is taken in the principal ray, the repreſentations of all lines, perpendicular to the picture, will converge to the point of ſight. And conſequ ntly the picture will appear a regular piece of perſpective, to an eye placed any where in that line.*

Cor.

Cor. 3. *Likewiſe by the ſame reaſoning, the repreſentations of all parallel oblique lines will tend to their accidental point, if the eye recedes in the line drawn thro' that accidental point.* Fig. 7.

Cor. 4. *Theſe things hold true in an inclined picture, ſubſtituting lines parallel to the principal ray, inſtead of lines perp. to the picture.*

PROP. X.

If a light be put in the place of the eye, and the ſhadows of any objects be projected on a plane parallel to the picture, placed behind the objects; theſe ſhadows will form a piece of perſpective ſimilar to that made on the picture, when the eye is placed in the point of view; and will have the ſame appearance.

For the image made by the ſhadows, is formed by rays drawn from the light thro' all the points of the objects, till they cut the parallel plane. And the image on the picture, is formed by the very ſame lines cutting the picture. Therefore theſe two images, upon theſe parallel planes, muſt needs be ſimilar; and will appear equal to an eye in the point of view.

Cor. 1. *The images of objects, formed by their ſhadows, on a plane; will make a regular piece of perſpective.*

Cor. 2. *The images formed, by the ſhadows of the ſun's light, make a piece of military perſpective.*

For the ſun's light may be ſuppoſed to come from an infinite diſtance.

Fig.

PROP. XI.

8. *If the back ſide of the picture* AC *be a plain reflector; and if the eye be placed at* O *in the principal ray* ES, *produced as far beyond the picture. Then the eye looking towards the reflector, will ſee all objects in the ſame places of the reflector; as an eye at* E *will ſee them in the picture.*

Let F be any point of an object, and draw FIG perpendicular to the picture cutting it in I, and make GI = FI; and draw SI from the point of ſight S. Then ſince both EO and FG are perpendicular to the picture, the point F ſeen by reflection from O, or directly from E will appear ſomewhere in the plane FEO, and therefore ſomewhere in the line SI, the ſection of it with the picture. But (Optics, B. II. Prop. II.) F is ſeen by reflection in the line GO, and F is ſeen directly in the line EF. But both theſe lines interſect at V in the line SI. For the lines GI = FI, and ES = OS, and SI perpendicular to them all. Therefore their interſection muſt be in the line SI. But in looking towards the reflector, with the back towards the objects; the right ſide will appear on the left, and the left on the right; being contrary to what they are when the face is turned the contrary way. Now what is proved of the point F, holds of all points of all objects whatever.

Cor. *Tho' all objects and all points of them appear in the ſame places by reflection as when ſeen directly, yet the face being turned the contrary way, what is on the right will appear on the left, and what is on the left will appear on the right hand.*

PROP.

Fig. 9.

PROP. XII.

In viewing any piece of perſpective of a country, city, &c. the further the eye is placed from it, in the principal ray; the longer the draught appears to be, and that in proportion to the diſtance of the eye.

I call that the length which is taken along the geometrical plane, as it runs directly from the picture.

Let AC be the picture, EA the principal ray, CV the horizontal plane. Let E be the point of view, for which the proſpect was drawn. Then 'tis plain CB repreſents the length CF, to the eye at E. Let the eye be removed to D, in the principal ray AED; then the ſame CB repreſents the length CV. But by ſimilar triangles, CF : AE : : (CB : AB : :) CV : AD; and therefore the apparent length of the draught is greater at D than at E, in proportion of AD to AE.

Cor. 1. *Hence in any given piece of perſpective, the further the eye is placed from the picture; the larger and more extended it ſeems to be; and the nearer, the leſs it appears.*

Cor. 2. *No perſpective view will appear exactly like the original, but when it is ſeen from the true point of view for which it was made.*

For when it is ſeen from a point nearer than the true point, it gives too ſhort a view; and if from a point further off, the view is too long. And in no poſition of the eye but one, can all lines come from the objects to the eye, to cut the picture in the ſame points. And therefore the ſituation of all objects will not be the ſame, when viewed from different points. Whence no view can appear ſo natural, as when ſeen from its own point of view.

And

Fig. 9. And then all rays coming to the eye will come in their true directions, and make an appearance exactly ſimilar to the original.

Cor. 3. *Tho' the objects ſeem greater when the eye is further off, yet they really appear under leſs angles.*

For in this caſe the reaſon of their appearing greater, is not becauſe the ſeveral parts of the view appear under greater angles, for they really appear under leſſer; but becauſe the view appearing longer, we judge the diſtance to be greater, and therefore the objects ſeem greater, and the view more extenſive. But this apparent greatneſs ariſing merely from judgment, it may be different in different people.

PROP. XIII.

If any print or perſpective view, be looked at thro' a lens, whoſe focal diſtance is equal to the principal ray, and the print placed in its focus; it will be ſo magnified, as to have the very ſame appearance, as the place it was drawn for.

10. Let L be the lens, AC the picture placed in its focus, whoſe focal diſtance is LS = the principal ray. Then (Cor. 10. Prop. XX. B. III. Optics), all the rays iſſuing from the picture AC, in its focus, and refracted by the lens L, will emerge parallel to the eye; therefore they come to the eye as they do from the objects at a great diſtance; therefore the view is magnified. Alſo (by Prop. XXXVII. ib.) the apparent magnitude of the view, or any part of it (that is the angle it appears under), ſeen thro' the lens; is the ſame as that ſeen by the naked eye from L, that is from the point of view of the print. But the apparent magnitude of the view or any part of it ſeen from L, is the ſame as that of the country itſelf (or other object) ſeen from

from L, ſince the ſame lines go to all the correſpondent points. Therefore the angles, the whole and every part of the print is ſeen under, thro' the lens, are the ſame as thoſe of the object or objects ſeen with the naked eye from L. And therefore both muſt neceſſarily have the ſame appearance. Fig. 10.

Cor. *Glaſſes which are not of a due focal length, will not give the exact appearance of a place. Shorter glaſſes make the diſtances leſs, and ſo contract the view. And longer glaſſes make the diſtances greater, and extend the view.*

For by Prop. XII. the further the eye is off, the larger and more extended the draught appears. And when 'tis ſeen thro' a lens of any focal diſtance, it appears equally large, as ſeen at that diſtance with the naked eye. Therefore it follows that a print ſeen thro' a lens of a long focus, appears larger than when ſeen thro' one of a leſſer focus.

Scholium.

Since glaſſes of a long focal diſtance, give a large and extenſive proſpect of a country, therefore they are better than ſhorter glaſſes. And when the proſpects are well drawn, and properly coloured; it is very delightful to view them thro' a good glaſs, as they ſo nearly imitate nature. And tho' there is but one focal length that will give a true appearance, yet the draught will always appear a regular piece of perſpective, tho' it may not exactly repreſent any place in the world, ſuppoſing the eye placed ſomewhere in the principal ray. And the draught will ſeem longer in proportion to the focal diſtance of the glaſs made uſe of; or in proportion to the apparent diſtance of the neareſt part of the picture.

As there is nothing more pleaſant than viewing the draughts of countries, towns, cities, magnificent

Fig. 10. cent buildings, and other grand objects, when well drawn: to see them to the best advantage, the focal distance of the glass should be just so long, as not to shew the scratches, and coarseness of the engraving; or not much longer; for then the view will be narrow, and the parts too small to be seen so far off. And if it be far off, it will hide the beauties as well as deformities. And to get a proper glass, observe at what distance the scratches disappear to the naked eye, and that is the focal length of the glass. Perspective views should be drawn, so that the point of view be further off, than is generally practised, if you would have the piece to be a true copy of nature. The principal ray should not be less than two foot, and then the draught being looked at thro' a lens of that focal distance, it will appear in perfection, and give a true representation of the place it was drawn for. The view should be so large as to subtend an angle at the lens of about 30 degrees. And it is proper to put the lens in a short square tube; which will confine the sight, and direct it to the perspective draught; which, to compleat its beauty, ought to be coloured with the same colours as the natural objects appear in. For which purpose water colours need only be used, as being sufficient for that purpose.

PROP. XIV.

11. *If* G *be any point in the geometrical plane,* GF *perpendicular to the ground line* DC, S *the point of sight, and* SF, EG *drawn. The representation of* G, *shall be at* P *in the line* SF; *and so placed, that* ES : SP : : GF : FP.

For since GF and ES are parallel, they are both in one plane, which plane cuts the picture in the line SF. And all lines as EG, drawn from E to any point

point of the perpendicular GF, will interſect the picture in the line SF; thus EG interſects it in P, and therefore P is the picture of G. Now the triangles ESP, GFP are ſimilar, for by the parallels, angle SEP = FGP, and the vertical angles at P are equal; therefore GF : FP : : ES : SP. Fig. 11.

Cor. 1. *If* G *be any point in the geometrical plane,* GF *perp. to the ground line, and* SF *drawn, and* EG *to interſect* SF *in* P. *Then* P *the image of* G *will be ſo placed, that the ſum of the principal ray and perpendicular* ES + FG : *to the line* SF : : *as the principal ray* ES : *to* SP *the diſtance of the image* P *from* S: *when the point* G *is beyond the picture. But when* G *is on this ſide the picture, you muſt take* ES — FG.

For ſince ES : SP : : GF : FP, therefore ES + GF : SP + PF (SF) : : ES : SP. And ES — *g*F : S*p* — *p*F (SF) : : ES : S*p*.

Cor. 2. *If the picture* AC *is oblique to the geometrical plane, the ſame proportions will hold good.*

This is evident, becauſe ES and FG will ſtill be parallel, and the triangles ESP, and GFP ſimilar.

Cor. 3. *Hence, if the given point be beyond the picture, its image will be above the ground line; if on the ſame ſide as the eye, it will appear below the ground line.*

PROP. XV.

The length of a line GF *ſtanding upright upon the geometrical plane, is to the length of its image gf in the picture; as the hight of the eye* ER, *to the diſtance of the image of its foot from the horizontal line, gn.* 12.

Let E be the eye, RCB the geometrical plane, ESTNV the horizontal plane. Produce GF and

Fig. 12. its repreſentation *gf*, to cut the horizontal plane at N and *n*, and draw NT parallel to VA. Then ſince GFN, and *gfn* are parallel, the triangles EGF, E*gf*, as alſo EGN, E*gn* are ſimilar, whence FG : *fg* : : EG : E*g* : : GN : *gn* : : ER : *gn*.

Cor. 1. *As the diſtance of the line from the eye, to the diſtance of its image from the eye; ſo the length of the line, to the length of its image.*

For by ſimilar triangles, FG : *fg* : : EG : E*g* : : DN : E*n*.

Cor. 2. *As the diſtance of the eye from the plane* TNG *(in which the line is), to the diſtance of the eye from the picture* ES, *ſo the length of the line* FG, *to the length of its image fg.*

For FG : *fg* : : EN : E*n* : : ET : ES.

Cor. 3. *The ſame is true on an inclined picture, provided the line* FG *be parallel to the vertical line* SX.

For the triangles EGF, E*gf* will be ſtill ſimilar; as alſo EGN, and E*gn*.

Scholium.

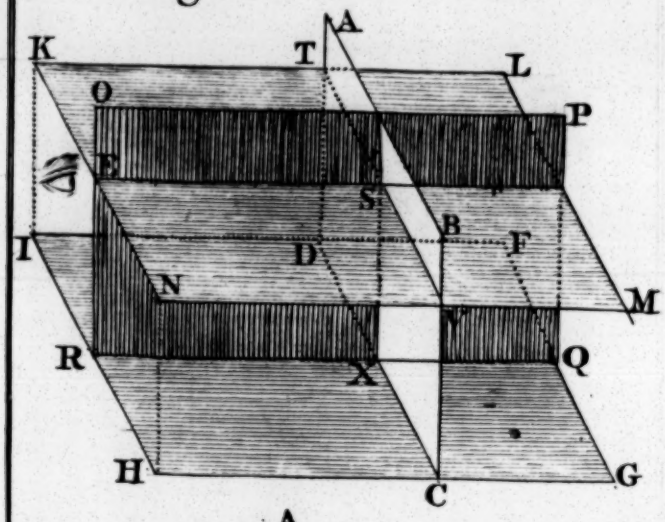
This propoſition is the foundation of all ſchenography. By its help all elevations are performed, the poſitions and hights of all ſolids truly laid down. And the ſchenographic repreſentations of all bodies exactly delineated.

PROP. XVI.

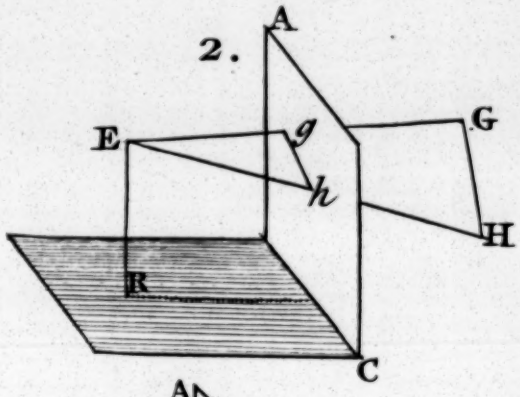
If a point be elevated above the geometrical plane, it is the ſame thing as if it was in a new geometrical plane raiſed ſo much higher, as is the hight of that point. The point of diſtance remaining the ſame.

13. Let E be the eye, S the point of ſight, AC the picture, IHQ the geometrical plane, and GF a line

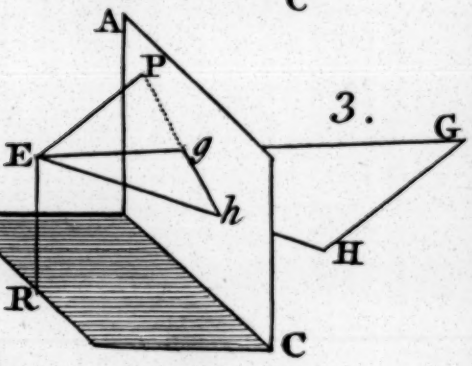
Fig. 1.



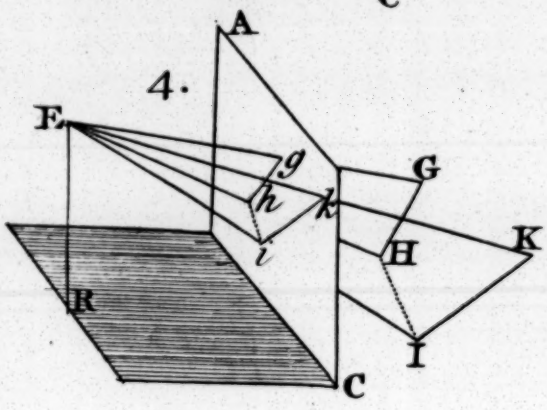
2.



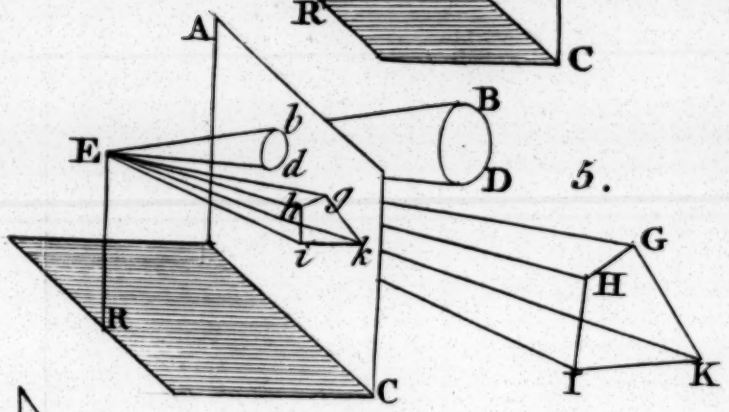
3.



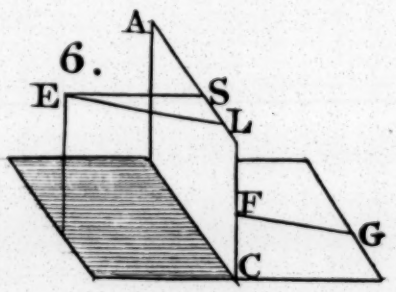
4.



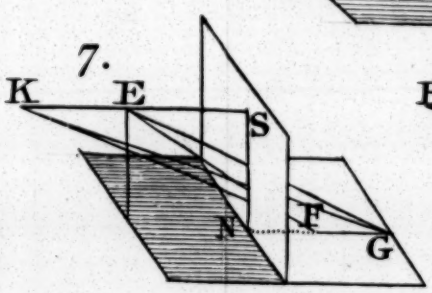
5.



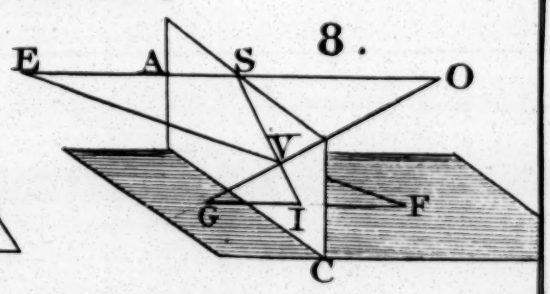
6.



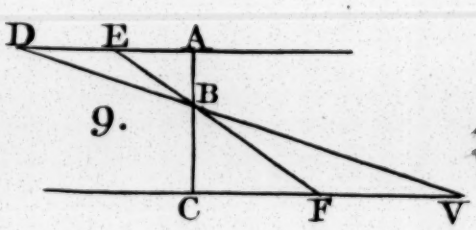
7.



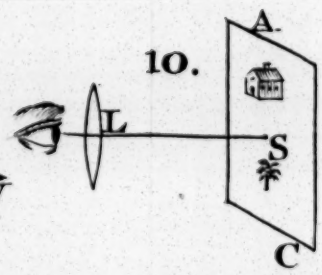
8.



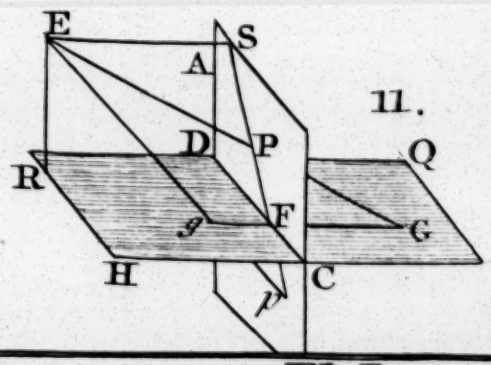
9.



10.



11.



line ſtanding perpendicularly upon it. Thro' the top F draw the plane KLMN, which will cut the picture perpendicularly in the line *dfc*, and the hight of the eye in *r*. Therefore if we make this new plane the geometrical plane, the point F will be a point in it. And the hight of the eye will be E*r*, and ground line *dc*. And the hight of the eye, and the picture will be ſhortened by the hight R*r* or FG or *fg*; while the point of diſtance, and principal ray ES remains the ſame. Fig. 13.

Cor. *And thus every point, elevated above, or depreſſed below, a given geometrical plane; may be aſſumed in ſome other geometrical plane, drawn thro' that point, parallel to the firſt.*

PROP. XVII.

If E *be the eye,* AC *the horizontal plane,* ER *the geometrical plane. And if any objects (ſituated in the plane* GR *perpendicular to the horizon) are to be drawn upon the plane* AC, *for the picture. They will appear in the ſame places, as when the hight of the eye, and the diſtance, are changed for one another.* 14.

For let G be a point to be repreſented in the plane SC. Draw the line EG to cut SC in *g*, and *g* will be its place in that plane; where ES is the hight of the eye, and SC its diſtance But this is the very ſame thing as if ER was the hight of the eye, and ES its diſtance, AC the picture, and RG the geometrical plane, in which the point G is ſituated. This will be evident, by turning the whole figure about, till RG becomes the baſe; for then E being the eye, ER is its hight, ES its diſtance, &c.

Cor. *Hence the ſame rules that ſerve for drawing upon a vertical plane, will alſo ſerve for drawing upon a horizontal one.*

Fig.

PROP. XVIII.

The ſhadow of a right line, upon a plane cauſed by any light, is alſo a right line.

For if a plane be drawn thro' the light, and the given right line, and extended to the plane, it will cut that plane in a right line (Geom. V. 3.); and it is evident this line of ſection is the ſhadow of the given line.

Cor. 1. *If the right line given be parallel to the plane, its ſhadow will be parallel to that line.*

For the line being parallel to the plane will be parallel to the ſection, (Geom. V. 11. Cor.).

Cor. 2. *If the right line go thro' the light, its ſhadow is only a point.*

Cor. 3. *The image of the ſhadow (of a right line upon a plane) will alſo be a right line.*

This is evident by Prop. II. conſidering the ſhadow as an object.

Cor. 4. *If ſeveral parallel lines be parallel to a plane, all their ſhadows will be parallel to one another and to theſe lines.*

This is plain by Geom. V. 8. and Cor. 1. of this.

Cor. 5. *And if the light be at a great diſtance; the ſhadows of any parallel lines, will be parallel to one another.*

For then the planes paſſing thro' theſe lines, and the light may be eſteemed parallel. And therefore (Geom. V. 11.) theſe ſhadows, or their ſections with any other plane, will be parallel.

PROP.

Fig.

PROP. XIX.

The images of all rays, proceeding either from the ſun or a ſmall light, tend to the image of that light in the picture.

For the image of the light is the image of one end of any ray proceeding from that light; and the ray being a right line, its image will be a right line, proceeding thro' the image of the light, or tending to it.

Cor. 1. *The image of the ſhadow of any point, is in the right line drawn thro' the image of that point, and the image of the light.*

Cor. 2. *The images of all bodies are terminated, by lines drawn thro' the image of the light, and thro' the images of the extremes of theſe bodies.*

PROP. XX.

If a right line FG *be parallel to any plane* PQ; 15.
the image ik of its ſhadow IK *upon that plane, (by any ſort of light), ſhall tend to the accidental point* O, *of that right line.*

For (by Cor. 1. Prop. XVIII.) the line FG and its ſhadow IK will be parallel; and (Cor. 1. Prop. IV.) the images of theſe parallel lines FG, IK, tend to the accidental point of FG, that is to O.

Cor. 1. *If a line be perpendicular to the picture, the image of its ſhadow, upon the geometrical or vertical planes, or their parallels, tends to the point of ſight.*

For then the line is parallel to both the geometrical and vertical planes.

Fig. 15. Cor. 2. *If a line be perpendicular to the vertical plane; the image of its ſhadow upon the geometrical plane, or the picture, or their parallels, is parallel to the ground line.*

For then it is parallel both to the geometrical plane, and the picture.

Cor. 3. *If the line be perpendicular to the geometrical plane; the image of its ſhadow upon the picture and vertical plane, and their parallels, is parallel to the vertical line.*

For then the line is parallel to the vertical plane and the picture.

PROP. XXI.

16. *If* L *be the image of the ſun in the picture, which is below the horizontal line* AS, *when the ſun is before the picture; or above it, when beyond. And if* LI, LN, *be drawn parallel to* AS, SX; *cutting the vertical line* SX *in* I, *and the horizontal line* AS *in* N, *and* SL *be drawn. Then if a right line be expoſed to the ſun's light; the image of the ſhadow, which ſuch a line, perpendicular to the picture, or vertical plane, or geometrical plane, or their parallels, caſts upon the ſaid plane, is parallel to* SL, *tends to* I, *or tends to* N, *reſpectively.*

1. The ſhadow of the principal ray ES will be LS. For L is the image of the ſun in the picture, and a line drawn thro' L and the eye at E, paſſes thro' the ſun; and therefore L is the ſhadow of E. But planes drawn from the ſun, thro' all the parallels to ES, will cut the picture AC in parallel lines; becauſe the ſun is at an infinite diſtance, and therefore theſe planes are parallel, and their ſections with the picture, parallel (Geom. V. 11); that is, the ſhadows of the lines perpendicular

lar to AC will be parallel to one another on the picture, and to SL. Fig. 16.

2. If ſeveral lines be perpendicular to the vertical plane EPQR; their ſhadows upon the plane will be parallel to one another, as appears from what is demonſtrated juſt now. But theſe ſhadows being taken for objects, their appearances in the picture (by Prop. IV. Cor. 1.) will tend to their accidental point, which is I. For the ſhadow of a line paſſing thro' E, parallel to AS, will fall upon LI (Cor. 1. Prop. XVIII.), and that ſhadow falling on the vertical plane, will be parallel to the other ſhadows on that plane, and alſo in the line EI. Therefore EI being parallel to all the ſhadows, I will be their accidental point.

3. Since L is the ſhadow of E, the ſhadow of ER perpendicular to the horizontal plane, will fall in the line GLN. And (Cor. 5. Prop. XVIII.) the ſhadows of all lines perp. to the geometrical plane will be parallel to RG, which is the ſhadow of ER. And ſince EN is parallel to RG, N will be the accidental point of RG, and of all the ſhadows, upon the geometrical plane. And therefore their repreſentations in the picture tend to N.

And the ſame is true of the images of the ſhadows upon any parallel planes; ſince theſe ſhadows are parallel to the former.

Cor. *If the ſun is in the plane of the picture, the appearance of the ſhadow of any line perp. to it, is parallel to a ray of the ſun falling on it. If in the plane of the vertical plane, they tend to* I. *If in the horizontal plane, they tend to* N. *And the ſame for their parallels.*

Fig.

PROP. XXII.

The image of the ſhadow, which a line perpendicular to any plane caſts upon that plane, tends to the image of the point of ſituation of the light upon that plane; that is, to the image of the point where a perpendicular from the light falls upon that plane.

17. Let D be any light, BF a line perpendicular to the plane KH, draw DI perp. to the ſame plane; then I is the point of ſituation of the light upon the plane KH. Then ſince BF, and DI are parallel, they will be in one plane, DBFI; and the ſhadow of BF will be in the ſame plane. But this plane cuts the plane KH in the line IFG. But the image of that line in the picture (by Prop. II.) is alſo a right line *ifg*. Therefore *fg*, which is the image of the ſhadow FG, tends to *i*, which is the image of I, the ſituation of the light.

Cor. 1. *If the ſhadow of a line perp. to a plane, fall on that plane and others parallel to it; the images of the ſeveral parts tend to the accidental point of any part.*

For theſe parts being parallel, their repreſentations (by Cor. 1. Prop. IV.) tend to the accidental point.

Cor. 2. *If many parallel lines ſtand at any inclination upon the plane* KH; *and if* DI *be drawn from the light* D *parallel to theſe lines, cutting the plane in* I. *Then the images of all the ſhadows will tend to the image of* I *in the picture.*

For the plain paſſing thro' DBF will ſtill cut the plane KH in the right line GFI.

PROP.

Fig.

PROP. XXIII.

The properties of an inclined picture ABC *are the same as thoſe of an upright one, in regard to the places, poſitions, and magnitudes of any points, lines or figures; taking* ER *for the hight of the eye, parallel to the vertical line* SX. *And inſtead of lines perpendicular to the picture or to the geometrical plane, taking the ſame lines parallel to* ES *or* SX. 18.

Let NG be a line in the geometrical plane parallel to the ground line XC; *ng* the image of it in the picture. Then the plane ENG, drawn thro' the eye at E, and the line NG which is parallel to the picture, will cut the picture in the line *ng* parallel to NG. And ſo would it do in the upright table, by Prop. III.

Let GF be a line ſtanding upon the geometrical plane at G, parallel to SX. Then *fg* its image will alſo be parallel to SX, or perpendicular to CX. And ſo it is in the upright picture, by Cor. 2. Prop. III.

Draw RG which will meet *fg* at I in the ground line CX. Then by ſimilar triangles, GF : *gf* : : GE : *g*E : : RG : RI : : RE : I*g*; which are the ſame proportions we have in Prop. XV. for the upright picture. Likewiſe NG : *ng* : : EG : E*g* : : ER : *g*I; which is the ſame porportion, as in the upright picture.

So that whether the picture and the lines ER, FG, ſtand ſloping, or upright upon the line CX, at the points R and G; the repreſentations of the lines NG, GF, will fall in the ſame places of the picture, and be of the ſame magnitudes in it.

Cor. *Hence if a perpendicular ſolid, as a priſm or cylinder, &c. as* DM, *is to be drawn; before this can be*

Fig. 18. *be done, by the rules for upright pictures, lines muſt be drawn from the top* D *to the geometrical plane, parallel to* SX, *as* Dm. *Then by having the length of* mD, *the image of* D *may be found; and then the image of* M, *and the ſolid compleated.*

Scholium.

Hitherto I have been laying down ſuch principles as are the foundation of the whole practice of perſpective; which being well underſtood, the reaſon of the ſeveral operations in the practice will eaſily be ſhewn and made intelligible.

And tho' objects may be drawn in perſpective various ways, either upon a plane perpendicular to the horizon, or parallel to it, or inclined in any given angle; yet ſince almoſt all objects ſtand perpendicular to the horizon, and all views and perſpective draughts, are hung againſt a wall, which is alſo perpendicular to the horizon. And ſince the rules for drawing are the moſt ſimple and eaſy, when the picture is in that poſition. Therefore I have given all the principles in full for that ſort of projection. And in order to extend the rules to other kinds, I have given ſome propoſitions, ſhewing how the other ſorts may be reduced to this.

Alſo as it is moſt natural to look at a draught of any thing directly, rather than obliquely. And all towns, cities, caſtles, towers, churches, edifices of all ſorts, ſtand in a perpendicular poſition, this ſort muſt be eſteemed the moſt natural of any, as well as the moſt ready, familiar, and eaſy.

And tho' the objects I have been deſcribing, are ſuch as fall below the horizontal line, and beyond the picture. Yet all the propoſitions are equally true, tho' the objects are never ſo high; or when they are between the eye and the picture. But to have drawn them ſo, would have required larger figures and more room.

And

And ſince ſhadows ſerve to diſtinguiſh the different ſides of a body, and to ſhew us what ſides are towards the light, and what from it, and thereby enable us to judge of the poſitions of all the parts of it; and upon this account they give life to a piece of perſpective, and are the chief ornament and beauty thereof. And as this light may either come from the ſun, whoſe rays are to be eſteemed parallel to one another, as coming from an infinite diſtance. Or they may proceed from a ſmall light as a lamp or a candle; and then they diverge from a point near hand. And ſince a perſpective draught is imperfect without ſhadows; therefore I have laid down a few propoſitions, ſhewing the direction and poſitions of the ſhadows of bodies upon all ſorts of planes, by any ſort of light; from whence the method of laying them down in perſpective is eaſily deduced. Fig.

SECT.

Fig.

SECT. II.

The Practical Rules for drawing in PERSPECTIVE.

HAVING in the firſt ſection treated of the theory or fundamental principles of perſpective; I come now to lay down the rules by which all the operations, in the practical part, are performed. All which are eaſily deduced and demonſtrated from the foregoing theory.

The nature and foundation of drawing in perſpective is eaſily underſtood, by looking thro' a glaſs window, and viewing the ſeveral objects without, and whilſt you keep your eye fixed ſteady in one place, imagine theſe objects painted on the glaſs in the ſame places they appear in; that is, in the places where the rays coming from the ſeveral objects to the eye, meet the glaſs. For if ſuch pictures of the ſeveral objects were actually deſcribed upon the window, it would be really a perſpective draught. But then it would only be drawn mechanically, without any rules of art. Whereas, by the practical rules, which will be here laid down, the very ſame draught may be made, without looking at the objects; and that from certain data, ſuch as the diſtance, poſition, altitude, &c. of the ſeveral objects to be deſcribed.

The practical part of perſpective is either direct or inverſe; the *direct* method is that by which the appearance of any given object is repreſented in the plane of the picture. And the *inverſe* method is that which finds out the object, by having its image given in the picture. In what follows we ſhall

ſhall ſuppoſe the picture directly oppoſed to the eye, and by the help of ſuch lines and points, as we have given, we ſhall find the appearance of any object upon that plane. In what went before concerning the demonſtrations, we were obliged to view it ſideways; that the eye might ſee all the interſections of the ſeveral lines and planes concerned therein; many of which, in any other poſture, would be hid from the eye; ſo that each of theſe figures is really itſelf a piece of perſpective. But theſe principles being eſtabliſhed, we ſhall have no further occaſion for it in that poſition; but ſuppoſe it placed directly before the eye, as it is to be ſeen from the point of view. Fig.

In the direct method, we find firſt how to place the image of a point in the geometrical plane, in the picture. Then the image of a line in the geometrical plane is found, by finding the images of the extreme points. Then the image of any plain figure is found, by finding the repreſentations of all the ſides. Thoſe of curve lines are found, by finding a good number of points, and drawing a curve regularly thro' them. The image of a line perpendicular to the geometrical plane, is eaſily found by having the point or baſe given. And then the image of a ſloaping line. The image of a ſolid is had by finding the image of its baſe, and then erecting perpendiculars at the angles, of a proper hight; and joining the tops of them by right lines.

In general, the images of all lines in the geometrical plane, which are perp. to the ground line, converge to the point of ſight. And thoſe that cut it obliquely tend to their accidental point. And the images of all lines parallel to one another, except ſuch as are parallel to the picture, converge to ſome point or other.

All

Fig. All lines parallel to the ground line, have their repreſentations on the table parallel thereto. And the images of all lines perpendicular to the geometrical plane, are perpendicular to the ground line. All theſe things will be exemplified particularly in what follows.

PROB. I.

To repreſent an object by military perſpective.

Military or orthographic perſpective is the drawing lines thro' all the points of the object perpendicular to the plane of the picture, placed in a proper poſition; and then connecting all the points by right lines as they are in the object. Here all lines that are not directly expoſed to the eye, muſt be made leſs than they are, ſo much as the obliquity is greater. Angles may appear either bigger or leſſer than the truth, according to the poſition of the plane of their ſides. All lines that are parallel in the object, muſt be parallel in the picture. And thoſe perpendicular to the horizon, muſt be perp. to the ground line.

The practice is eaſy. Draw any line for the ground line, and the object being placed, or ſuppoſed to be placed, in a convenient ſituation; let fall perpendiculars from every point or angle of it upon the plane of the picture; drawn in obſcure lines; all theſe will be perpendicular to the ground line. Then lines muſt be drawn from one point to another as they are in the object.

Or thus, Draw the plan or baſe of the object, and draw from all the angles, the ſeveral altitudes of the parts of the object in theſe places, parallel to one another; and perpendicular to the ground line, all in obſcure lines. Then join the tops of theſe lines, by drawing other lines, as they are in the object, and all lines belonging thereto that are viſible;

viſible; rubbing out thoſe that are hid from the eye. But ſtrictly ſuch lines as are not directly oppoſed to the eye muſt be diminiſhed, according to the ſine of the angle of incidence, from the eye. And the angles will be increaſed or decreaſed, in conſequence of that obliquity. Laſtly, the ſeveral parts or planes compoſing the ſurface of the body, that lie from the eye muſt be ſhaded; the more, the more oblique they are. Fig.

Sometimes you muſt draw the front or profile as well as the plan, to ſhew the ſeveral altitudes.

Examp. 1.

Let OPQRS be a piece of fortification ſeen in front. ABEF is the plan, where all the lines BA, CD, &c. perpendicular to the ground line BE, are reduced in length in the proportion of *a*B to AB, by reaſon of the obliquity of the rays, upon the lines AB, CD, in the baſe. GHIKLMN is the front, where all the lines H*b*, I*i*, &c. are ſhortened in proportion of the radius to the ſine of inclination of the rays upon the lines H*b*, I*i*, &c. in front, or plane GLMN. Then the figure OV*l*RS, being made equal to GHLMN, and the lines OP, V*v*, &c. and RQ, be made equal to BA, CD, &c. and EF; and the lines joining on the ſeveral planes be drawn, as P*v*, OV, *l*R, &c. the ſolid will be formed; which being ſhaded, gives the true appearance thereof OPQRS, according to this method of drawing. 19.

Ex. 2.

ABCD is a table, ſeen both in front and profile. Let EC be the ground line, EF a line parallel to the front. MG the length of the table, MF the length contracted by the obliquity of the lines in the front, parallel to the ground line; and MK perp. to MF, is the breadth; and MH the contracted 20.

Fig. 20. tracted breadth, by the obliquity of the lines perp. to the line EF. Make HI = HK, and draw IN parallel to MF, and make ML = MH, and draw FN parallel to ML; then LNFM is the plan. Make the figure OCDP equal and ſimilar to LMFN. And on the points O, C, D, P, raiſe the perpendiculars OQ, CR, DS, PT, equal to the contracted hights of the table by the obliquities of the lines OQ, CR, &c. Draw a plane ABV, thro' the points Q, R, S, T, whoſe ſides ſhall be parallel to QR, RS, ST, TQ; and finiſh your table as in the figure.

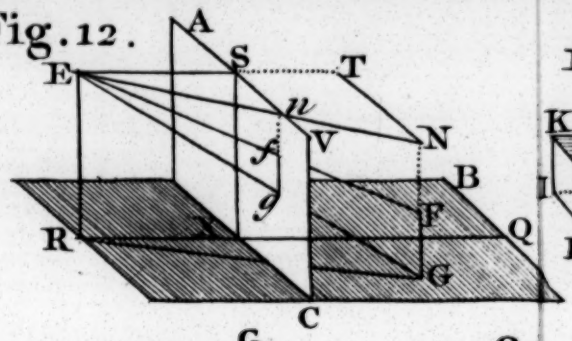
Scholium.

Drawing by this ſort of perſpective, is no more than drawing the ſhadows of all the lines of an object, which the ſun, (or a light at an infinite diſtance), caſts upon a plane; and then ſhadowing the parts.

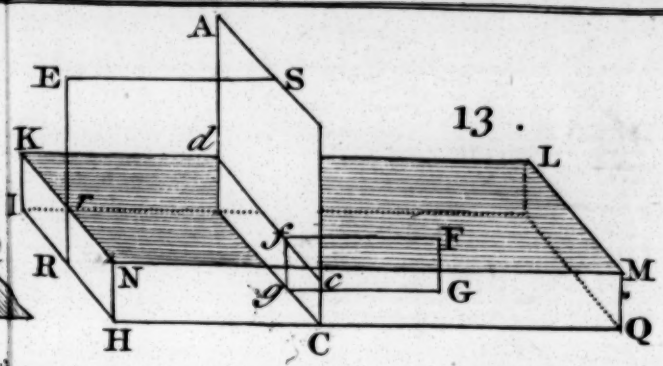
The point of ſight in this kind of perſpective is any where. For the eye is ſuppoſed to project any parts of the object, by lines perpendicular to the picture; which comes to the ſame thing as viewing them from an infinite diſtance.

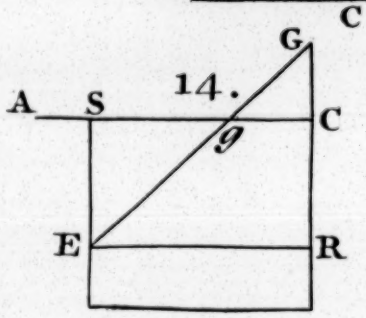
This ſort of perſpective is the beſt for drawing ſmall bodies, as things within a houſe; and particularly all ſorts of engines; for ſtanding at a diſtance, the ſides that are parallel in the object, ſcarce appear otherwiſe than parallel. And upon account of its eaſineſs, it is an excellent method to learn to draw by, and very proper to be practiſed by young ſtudents that are learning to draw; for by this they acquire a habit of judging of the poſitions of the ſeveral parts of an object, and by that means readily learn to draw by hand. The principal difficulty is to know how much any line, (or any other parallel to it) is contracted in the draught, and to do this, either the angle which the viſual ray makes with that line muſt be given; or it muſt be found by

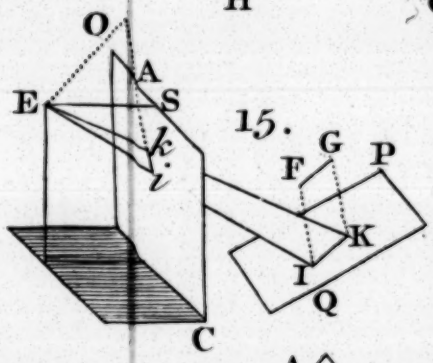
Fig. 12.

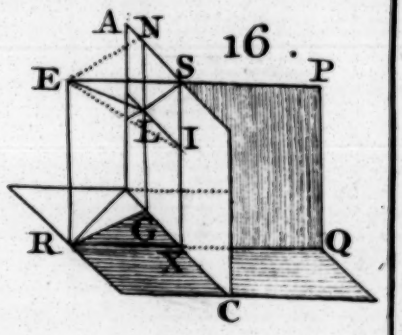


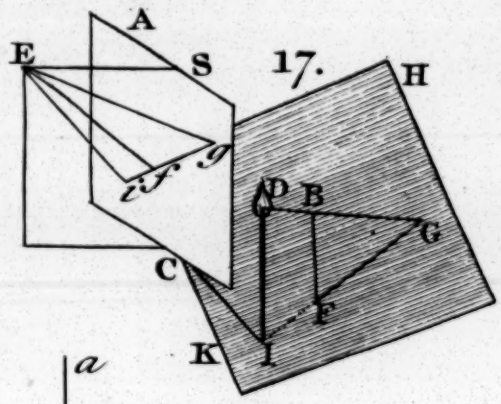
13.

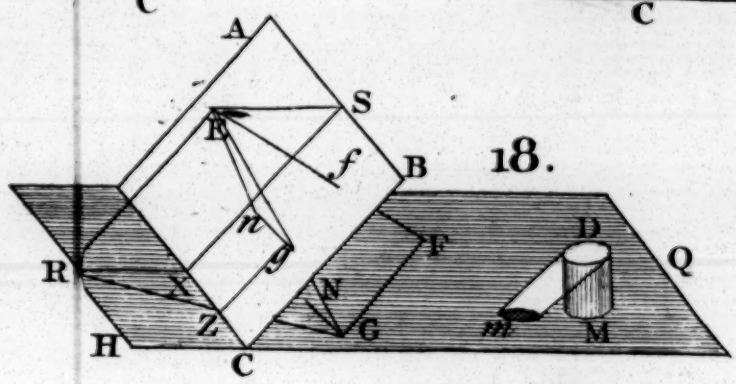


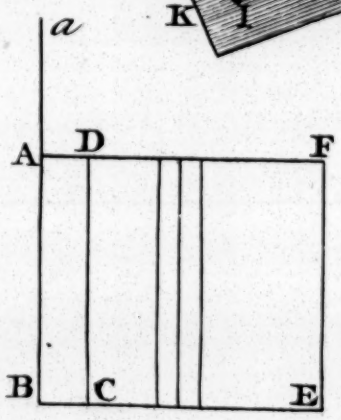




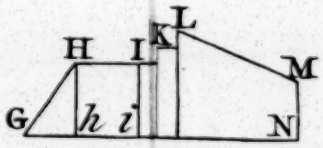


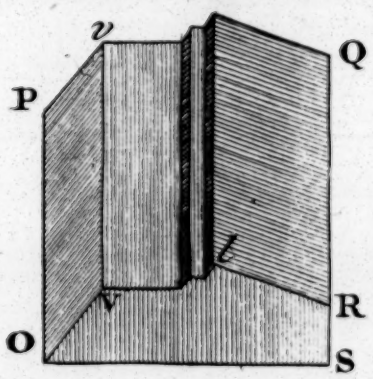


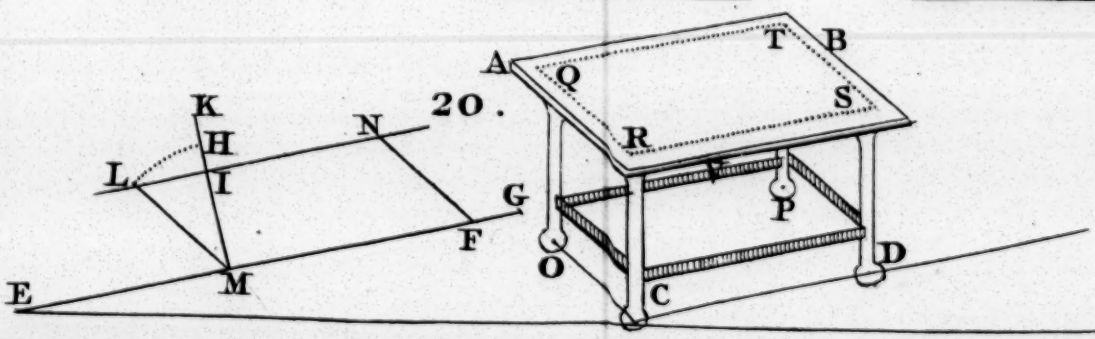




19.







Perspective

Pl. II. *pa* 32.

by experiment; and these being had, the thing will be easy. But it must be observed, that the method here used of laying down the plan, is not accurate, but only an approximation; and is used only to avoid a more tedious and troublesome construction. And therefore it gives not the angles exactly true. But for such as would be exact in finding the true representation of an angle, I shall lay down the following problem. Fig. 20.

PROB. II.

To find the apparent magnitude of an angle in military perspective.

Let ABCD be the plane, BAD the given angle in that plane. At the point A erect the perpendicular AP, to that plane; thro' AP and the eye (at an infinite distance) draw the plane PAFG, which will be perp. to the given plane AC. Let the angle AGF be the elevation of the eye above that plane, and let AF be perp. to FG. Then thro' AF and FG draw the two planes AFE, and FGBE, both perp. to AFG; then since the three planes AFE, AGB, FGB, are all perp. to the plane AFG, their intersections will be all perp. to AFG, and consequently parallel to one another; and therefore FE parallel to GB. Therefore if a ray passing thro' F fall upon G, another ray parallel to it falling on B will pass thro' E making FE = GB. Therefore it is evident, the angle BAG in the plane ABCD, is projected into the angle EAF in the perpendicular plane EAF. And the angle BAG is the declination of the plane AFG (or of the eye) from the side BA. Therefore in the triangle AFG, rad : S.AGF :: AG : AF = $\frac{AG}{rad} \times$ S.AGF, and in the tri- 21.

Fig. 21. triangle AGB, rad : tan. GAB : : AG : GB = $\frac{AG}{rad} \times t.GAB$; therefore in the triangle AFE, rad : tan. FAE : : AF : FE or GB : : $\frac{AG}{rad} \times$ S.AGF : $\frac{AG}{rad} \times t.GAB$: : S.AGF : tan. GAB.

Cor. 1. *Hence, as ſine of elevation of the eye* AGF *above the plane* ABCD : *radius* : : *tan. declination* BAG *of the eye or vertical plane from the ſide* BA : *tan. of the apparent angle* FAE, *from the ſame vertical plane* AFG.

Cor. 2. *After the ſame manner that the angle* FAE *is found for the repreſentation of the angle* GAB *on one ſide of the vertical plane* AFG; *the repreſentation of the angle* GAD *on the other ſide, is found; and conſequently the whole angle* BAD.

PROB. III.

Any figure or figures being given in the geometrical plane, to lay them down in a proper ſituation for drawing them in perſpective.

22. Let AB be the ground line, CDEF the given figure. From every point of the figure, C, D, E, F, draw perpendiculars to the ground line CG, DH, Ff, EK, and continue them till Gc = GC, Hd = HD, If = IF, and Ke = KE. Then draw *cd*, *de*, *ef* and *fc*. And the figure *cdef* is in a proper ſituation for drawing.

For if the figure CDEF was to remain in its proper place, the projection of it in the picture would interfere with the figure itſelf, and one of them would confound another. Therefore the original

ginal is transferred to the other ſide of the ground line in an inverted poſition, as if the whole figure was turned upſide down over the line AB into the poſition *cdef*. And this is juſt as it would appear by reflexion in a looking-glaſs; and therefore the inverted figure, will by reflexion ſhow the true figure. And as the diſtances of the ſeveral points *c*, *d*, *e*, *f* are the ſame as of the points C, D, E, F, and in the ſame perpendiculars to the ground line; therefore they anſwer exactly the ſame ends, whether the true or inverted figure is made uſe of. Fig. 22.

Scholium.

Hence in what follows, all figures placed in the geometrical plane, are ſuppoſed to be tranſpoſed to the other ſide of the ground line, at the ſame diſtance, and there drawn in an inverted poſition. But it is not always neceſſary that the figures be actually drawn; it is ſufficient in many caſes, to have only the diſtances of their ſeveral points from the ground line, and the points where the perpendiculars cut it. For by having theſe things, all the operations may be performed as well as if the figures were drawn.

PROB. IV.

To find the vaniſhing line of any plane, whoſe interſection and inclination to the picture are given.

Let S be the point of ſight, PQ the interſection of the plane with the picture. Draw RSP perpendicular to PQ, and SE parallel to it, and equal to the principal ray. Make the angle SER equal to the compliment of the angle, which the given plane makes with the picture, interſecting RP in R; draw RV parallel to PQ, for the vaniſhing line of the plane. 23.

Fig. 23. The vaniſhing line is the interſection of the picture, and a plane paſſing thro' the eye parallel to the given plane. Juſt as the interſection of the picture and the horizontal plane, is the vaniſhing line of the geometrical plane.

For ſuppoſe the plane RES to be raiſed up perpendicular to the plane of the picture. Then as E is the eye, and RV parallel to PQ; a plane paſſing thro' ERV, will then be parallel to the plane paſſing thro' PQ; and inclined to the picture in the angle ERS. Therefore RV is the vaniſhing line ſought.

PROB. V.

24. *To find the accidental point of a right line; having given its elevation on the geometrical plane, and the angle its baſe makes with the ground line.*

I call that its baſe, which is a line drawn from the point where it interſects the geometrical plane, to the point where a perpendicular interſects the geometrical plane, which is drawn from the top of it.

Let S be the point of ſight, SP the horizontal line, XQ the ground line. Then as it is all one where the line ſtands, if it be but in a parallel poſition; therefore ſuppoſe it cuts the geometrical plane at X. Make the angle BXC the declination of the baſe, or the angle it makes with the ground line, and DXF its elevation above the plane; which here is towards the eye. Let SE, equal to the principal ray, be perp. to SP, and make the angle SEP = BXC the declination; thro' P, draw PQ parallel to the vertical line SX. Then make PG = PE, and the angle PGR = DXF the depreſſion, here below the geometrical plane; and R is the accidental point.

For

For imagine the plane ESP to be raiſed up till it be perpendicular to the plane of the picture SXQP, then the angle SEP being equal to BXC, the plane paſſing thro' EPQ will be parallel to the plane in which the given line is, whoſe baſe is XC; therefore PQ is the vaniſhing line of that plane. Then ſince PG or PE is the diſtance of the line PQ from E; and the angle PGR being equal to DXF, therefore ER muſt be parallel to DX (ſuppoſing ER to be right over S, as was ſaid before); and therefore R is the accidental point of DX, or of the given line, which is parallel to it, by Prop. VIII. Fig. 24.

It is eaſy to know, from the poſition of the given line, whether the angle SEP is to be taken to the right or to the left; and whether the angle PGR is to be taken upwards or downwards.

Cor. 1. *If the given line have no declination, its accidental point is in the vertical line* SX.

Cor. 2. *If the given line be parallel to the geometrical plane; its accidental point is in the horizontal line* SP.

Cor. 3. *Two lines proceeding from one point, make the ſame angle, as two lines drawn, from the eye to their accidental points do.*

PROB. VI.

A point in the geometrical plane being given; to find its appearance in the picture.

1. Let AB be the ground line, DD the horizontal line, S the point of ſight; D, D, the points of diſtance. And let C be the point given, tranſpoſed to the other ſide of the ground line. Draw CF perpendicular to AB; make FG = FC, draw FS, then draw GD to the oppoſite point of Fig. 25.

Fig. 25. distance D, if the original point is beyond the picture, to cut SF in *c*, then *c* is the image of C in the picture.

But if the original point be on this side the picture, you must draw a line from G to the point of sight on the same side; and then the intersection with SF, gives the image of C below the ground line.

For the triangles D*c*S and F*c*G are similar, whence DS : S*c* : : FG : F*c*. But (by Prop. XIV.) the image of C is in the right line SF, and so placed, that the principal ray : S*c* : : distance of the point : F*c*. But DS is equal to the principal ray (by Def. XVI), and FG is equal to FC the distance of the point. Therefore *c* is the image of C.

2. *Otherways thus.*

Set off the distance FC both ways from F to G and A, and draw from A and G, two lines to the opposite points of distance D, D; and their intersection *c* is the place of the image, if C is beyond the picture. Otherwise draw lines to the points of distance on the same side.

For since AF = FG, and DS = SD, therefore DG and DA intersect SF in the same point *c*, and consequently they intersect one another in the very same point.

3. *Or thus.*

When the distance of the eye, or the distance of the point, is very great, or when the point D or the point G, falls without the picture. Take any aliquot part of SD from S, and the same part of FC or FG from F; and from these points draw a line to intersect SF, which intersection will still fall upon the same point *c*, because similar parts are proportional to the wholes.

4. *Other-*

Fig.

4. Otherwiſe.

Let SD be the horizontal line, AB the ground 26.
line, S the point of ſight, C the given point tranſpoſed. Draw SE perpendicular to SD, and equal to the principal ray; and draw FC perp. to AB. Draw SF, and EC to interſect it in *c*, the image ſought. If the given point be on this ſide the picture, FC muſt be taken upwards.

For the triangle ES*c*, and CF*c* are ſimilar, whence SE : S*c* : : CF : F*c*. Therefore (Prop. XIV.) *c*, in the line SF is the image of C. Hence alſo, SE : E*c* : : CF : C*c*.

5. Otherwiſe thus.

Let SD be the horizontal line, S the point of 27.
ſight, AB the ground line, C the given point tranſpoſed. Draw SE perpendicular to SD, and equal to the principal ray. Draw the line EC, then draw any line CB and EL parallel to it, draw LB to interſect EC in *c*, the image of C.

If the given point be on this ſide the picture, draw FC upwards.

For draw CF perpendicular to AB. Then by the ſimilar triangles ESL, CFB, and E*c*L, CB*c*; we have, ES : CF : : EL : CB : : E*c* : C*c* : which comes to the ſame thing as the 4th method.

6. Or thus.

From the point C tranſpoſed, draw two lines CB, CG, to the ground line AB; and two lines EL, ED parallel to them, from the eye at E, to the horizontal line LD. Draw LB, DG to interſect in *c*, which will be the image of C.

For by the ſimilar triangles, the lines LB, EC interſect one another in the ſame point, in which DG, EC interſect. And therefore LB, DG interſect in the ſame point.

Fig.

7. Otherwiſe.

28. Let SD be the horizontal line, S the point of ſight, AB the ground line, C the given point tranſpoſed, CF its perpendicular. Draw from any point D, two lines DB, DG, making any angle; make DG = principal ray, and EG = CF; draw GB, and EK parallel to it, and KC parallel to AB, cutting SF in *c*, the repreſentation of C.

For by the parallels, DE : EG : : DK : KB : : S*c* : *c*F, according to Prop. XIV. therefore *c* is the image of C.

Cor. 1. *From hence the accidental point of any right line may be found as follows.*

29. Let DE be the horizontal line, S the point of ſight, AB the ground line; then ſince the poſition of the line is given, its interſection with the geometrical plane, and alſo with the picture, is given. Let P be the point in the geometrical plane tranſpoſed, and I that in the picture. Find the appearance of P in the picture as at *p*. And thro' I, *p*, draw EI*p*F. Make *as* the diſtance of the foot of the line from the picture, PC : to the principal ray DS : : So is *p*I : to *p*K; and K is the accidental point of the given line.

30. For let PI be the given line, and let *p* be the appearance of P, in the picture; thro' *p* and I, and the eye at D, draw the plane DIP*p*, in this plane draw DK parallel to PI, then it is evident (Def. XVII.) K is the accidental point of PI. But the triangles P*p*I and D*p*K are ſimilar, whence *p*K : *p*I : : D*p* : *p*P. Let the plane DIPK revolve about the line DP, till it coincide with DS, then the plane DSCP is perp. to the picture, and PC perp. to the ground line; and the triangles D*p*S and P*p*C are ſimilar, whence D*p* : *p*P : : DS : PC. Therefore *p*K : *p*I : : DS : PC.

Cor.

Cor. 2. *Let* Ip *interſect the horizontal line in* E, *and the ground line in* F. *Then if* pF : pE : : pI : pK, K *will be the accidental point of* PI. Fig. 30.

For the triangles S*p*E, C*p*F are ſimilar, therefore DS : PC : : S*p* : *p*C : : E*p* : *p*F, and (Cor. 1.) DS : PC : : *p*K : *p*I; whence *p*E : *p*F : : *p*K : *p*I.

Scholium.

To avoid confuſion with many lines, if two threads be fixt at the points of ſight and diſtance, and be extended to the diviſions of the ground line, their interſection will give the points in the picture. It may be ſufficient only to apply the threads.

PROB. VII.

Any right line in the geometrical plane being given; to find its appearance in the picture.

Find the image of one end of the given line (by the laſt Prob.) and in like manner find the image of the other end of it. Then a right line drawn from one of theſe images or points to the other, will be the image of the given line.

Ex. 1.

Let FG be the given line tranſpoſed; find the image of F at *f*, and the image of G at *g*; then draw *fg*, which is the repreſentation of FG. Fig. 31.

Ex. 2.

Let the given line FG be parallel to the ground line PQ, draw FP, GQ perpendicular to the ground line. Then find the image of F at *f*, and of G at *g*. And draw the line *fg* for the image of FG. Fig. 32.

Or ſhorter thus, draw SP and SQ, and having found the point *f* as before, draw *fg* parallel to PQ,

to

Fig. 32. to cut SQ in *g*. Then draw *fg* for the image of FG. For (by Prop. III.) the image *fg* will be parallel to FG or to PQ. And (by Prop. XIV.) it will be terminated in the line SQ.

PROB. VIII.

A right lined figure in the geometrical plane being given; to find its repreſentation in the picture.

Find the appearances of all the angular points thereof in the picture (by Prob. VI.) and draw lines from one to another, as in the original.

Or find the appearances of all the lines that compoſe it, one after another (by Prob. VII.) and that will give the image of the whole figure.

And here all lines, perpendicular to the ground line, muſt have their images drawn to the point of ſight; and all lines parallel to the picture, will have their repreſentations parallel. By this means ſeveral ſhort methods of working, eſpecially in regular figures, will appear of themſelves.

Ex. 1.

33. Let NOPR be a ſquare, whoſe ſide NR is parallel to the ground line AB. DE the horizontal line, S the point of ſight. Make CF = CN, and CA = CO. OC and PI being perp. to AB. From S draw SC, SI, and FE to interſect it in *n* and *p*; draw *nr*, *op* parallel to AB, and *nopr* is the image or repreſentation of the ſquare NOPR.

For ſince CN = CF, *n* is the image of N; and CA being = CO = IP = IF, therefore *p* is the image of P, and therefore *nopr* is the image of NOPR.

Ex. 2.

34. Let LMNOP be a regular pentagon, DE the horizontal line, S the point of ſight, AB the ground line.

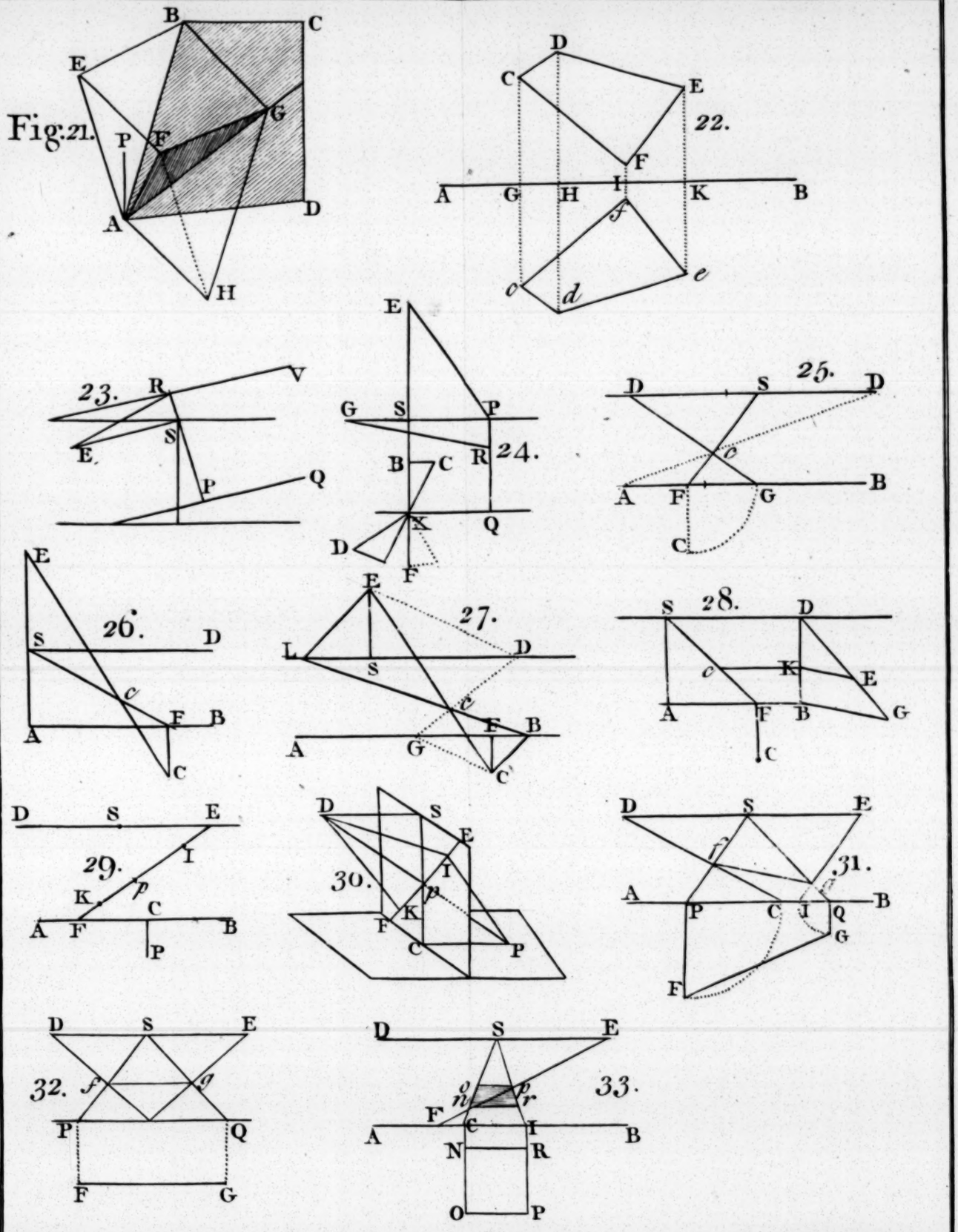


Fig. 21.
B C E G P F A D H
22.
D C E F A G H I K B f e d
23.
R V E S P Q
24.
E G S P B C R Q D X F
25.
D S D c A F G B C
26.
E S D c A F B C
27.
E L S D c A G F B C'
28.
S D c K E A F B G C
29.
D S E I K p C A F B P
30.
D S E I p K F C P
31.
D S E f A P C J Q B G F
32.
D S E f g P Q F G
33.
D S E n o r F A C I B N R O P

line. From the ſeveral angles L, P, O, N, M, Fig.
let fall perpendiculars upon the ground line, L*f*, 34.
P*g*, M*h*, O*i*, and N*k*. To the point of ſight S draw the lines *f*S, *g*S, *h*S, *i*S, *k*S. Make *f*1 = *f*L, and draw 1D, to the point of ſight D, cutting *f*S in *l*, the image of L. Make *g*2 = *g*P, and draw 2D cutting *g*S in *p*, for the image of P. Make *h*B = *h*M, and draw BD, cutting *h*S in *m*, for the image of M. Make *k*3 = *k*N, and draw 3E, cutting *k*S in *n*, the image of N. Make *i*4 = *i*O, and draw 4E, cutting *i*S in *o*, the image of O. Then draw the lines *op*, *pl*, *lm*, *mn*, *no*, which makes the image of the pentagon, required.

Ex. 3.

Suppoſe the two ſquares O, P, ſeen corner wiſe, 35.
are to be drawn, joining upon the ground line AB. S is the point of ſight, D one point of diſtance. From all the angles of the ſquares let fall perpendiculars upon the ground line AB; from the points of interſection draw lines to the point of ſight S. Make A*t* = *tt*, and from the points A, *t*, *t* draw lines to the point of diſtance D, to interſect the former; the firſt in *i*, *k*; the ſecond in *l*, *m*, the third in *n*. Then *iklt*, and *lmnt* are the images of O and P.

Shorter thus. Since the ſides of the ſquares make half a right angle with the ground line; their repreſentations will tend to the points of diſtance, on each ſide. Therefore from the angular points *t*, *t*, draw lines to the point of ſight D, and alſo to the point of ſight on the other ſide; theſe lines by their interſections will give the angles of the ſquares at *i*, *k*, *l*, *m*, *n*.

Or thus. Draw *t*S, *r*S, *t*S, and draw *t*D to interſect theſe in *l*, *m*. Thro' *l*, *m*, draw parallels, *nli*, *mk*, to the ground line AB; and making *li* = *ln*; this gives the angles *i*, *k*, *l*, *m*, *n*.

The

Fig. 35. The reaſon of theſe operations, is evident; for $tM = tt$, and $tK = tA$, and $rL = rt$, &c. Alſo ſince the lines ILN, and KM, are parallel to AB; *iln*, and *km* will alſo be parallel to AB.

Ex. 4.

To draw a floor of ſquares ſeen fore right.

36. This may be done either with or without a geometrical plan. Let ABGH be part of the plan, where the ground line AB is divided into the equal parts A*f*, *fg*, *gh*, &c. equal to the ſides of the ſmall ſquares. To the point of ſight S, draw AS, *f*S, *g*S, *h*S, &c. to BS. Then from A draw to the oppoſite point of diſtance, the line AE, which will cut the lines *f*S, *g*S, *h*S, &c. in ſeveral points, 1, 2, 3, thro' which, as many parallels muſt be drawn to the ground line AB; the laſt whereof is CF; in which all the diviſions are equal between C and F, and the ſame equal diviſions may be continued from C to K, and from F to I. And from theſe points of diviſion other lines may be drawn from S, which upon the line AE (going thro' the point of diſtance E) give other points, thro' which more parallels to the ground line are to paſs, and thus the whole ſpace ABIK may be filled with ſquares. And if more be deſired, draw from the point K to the oppoſite point of diſtance E, the line KE, which will cut all the lines drawn from S, in as many points, thro' which new parallels muſt be drawn to the ground line AB, which laſt is LN, paſſing thro' the point L, at the interſection of SB, KE.

For ſince $fA = fi$, the point 1 repreſents *i*, and *g*A being equal to *g*K, 2 repreſents *k*; and as *h*A $= hl$, 3 repreſents *l*, and ſo on. And the ſides of the ſquares being parallel to AB, their repreſentations in the picture, paſſing thro' 1, 2, 3, &c. will be parallel to AB, by Cor. 1. Prop. III.

If

If a line be drawn from *f* or *g* or *h*, &c. to E; that line will paſs thro' all the angles of the ſquares, as well as the line AE, which paſſes thro' the angles 1, 2, 3, &c. For ſuch a line cuts the lines drawn from S to the ſeveral diviſions of AB, in the angular points. As AE paſſes thro' all the angular points; therefore 1E or 2E or 3E (which are parts of the ſame line) alſo paſſes thro' all the angular points. Therefore a line paſſing thro' any diviſion in the firſt parallel, or thro' any diviſion in the ſecond, or in the third, &c. will alſo paſs thro' all the angular points. And therefore a line drawn thro' F and E, or thro' C and E, or thro' K and E, will alſo paſs thro' all the angular points of the ſquares. Fig. 36.

Ex. 5.

To draw a floor of ſquares ſeen cornerwiſe.

This alſo may be done either with or without a geometrical plane. Let ABGH be a part of a ſquare ſeen fore right, which contains the little ſquares. Here AB is divided into equal parts by the diagonals of the ſmall ſquares. Let AB be the ground line. DE the horizontal line, D and E the points of diſtance, S the point of ſight. Draw AS and BS to the point of ſight, and from all the points of diviſion in AB, draw lines to D and E the two points of diſtance, of which the extreme rays AE, BD, will cut the former in C and F; draw CF. Then ABFC is the image of a ſquare which contains all the little ſquares; and the other lines drawn to D and E, will by their interſections form the images of the ſmall ſquares, ſeen angle wiſe. And to fill up the whole, divide CF into the ſame number of equal parts as AB is divided into; and draw lines from D and E thro' all the points of diviſion, which will form the remaining ſquares. 37.

For

Fig. 37. For ſince the ſides of the ſmall ſquares, lying on the geometrical plane make half a right angle with the ground line, their appearances in the picture will tend to the points of diſtance, as their accidental points, by Cor 3. Prop. IV. And ſince AB is divided into equal parts by lines drawn from D or E; therefore all lines parallel to AB, and paſſing thro' the angular points, of the ſquares, will be divided into equal parts by the ſame lines; and therefore EF is divided equally, by the lines drawn from D and E; to the equal diviſions in AB continued.

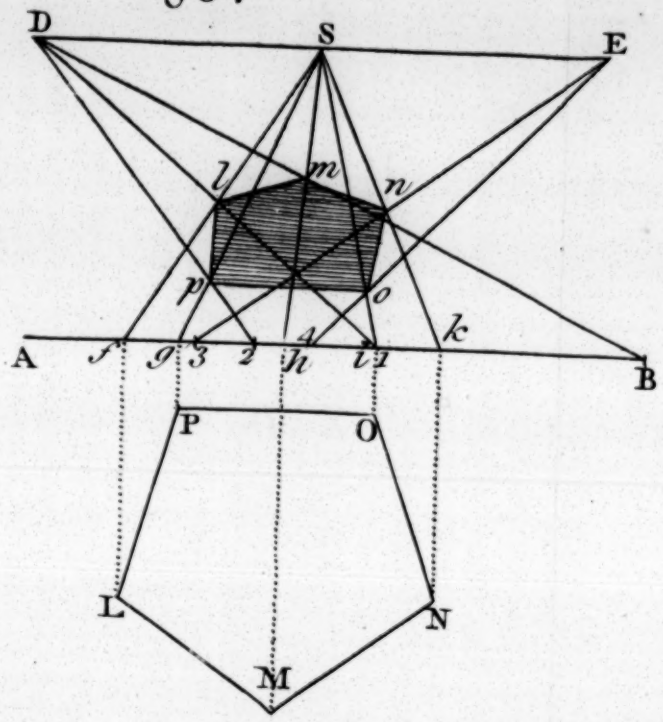
Ex. 6.

To repreſent a floor of equilateral triangles.

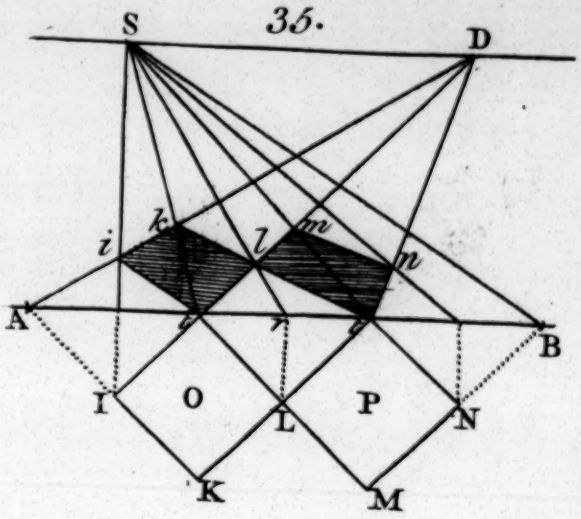
38. Let AB be the ground line, DE the horizontal line, S the point of ſight; D, E, the points of diſtance. Upon the horizontal line DE, make an equilateral triangle PIQ; ſo that SP and SQ may each of them be half the ſide of the triangle. To the point of ſight S, draw AS, BS. And from A and B draw to the points of diſtance E and D, the lines AE and BD, interſecting the former in C and F. Then ACFB is a ſquare in which the equilateral triangles are to be repreſented. Let ABGH be part of the ſquare upon the geometrical plane. AB being divided into equal parts, each equal to the ſides of the triangles; from all the points of diviſion, draw lines to the point P, and from the ſame points of diviſion, draw lines to the point Q, and thro' the points of meeting draw lines parallel to AB. All theſe lines by their interſections with the former, will make the repreſentations of the triangles in the picture, as required. And to fill up the ſquare, divide CF into the ſame number of equal parts, as AB is divided into; then drawing lines from P and from Q thro' all the points of diviſion; the ſquare ABFC will be filled up with equilateral triangles.

For

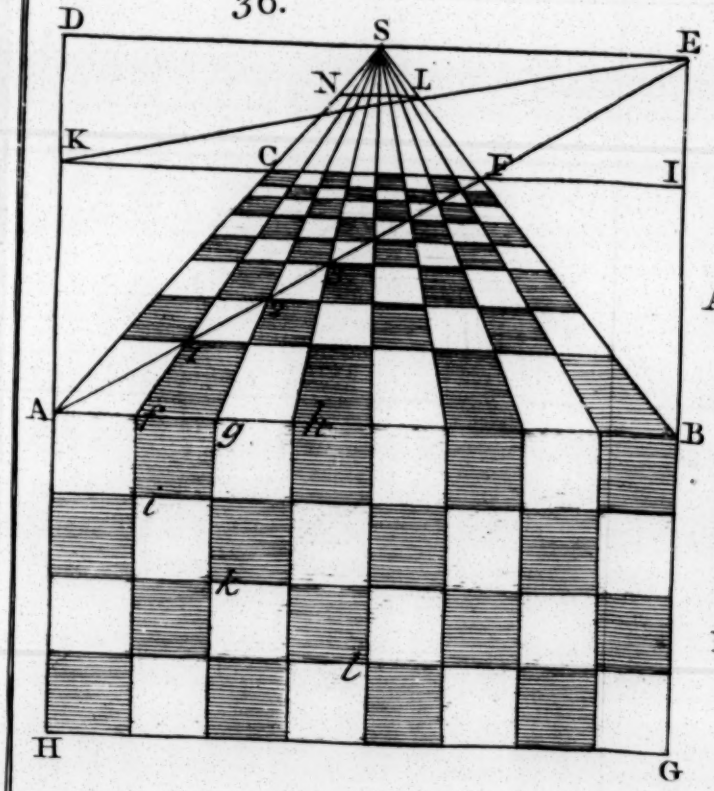
Fig. 34.



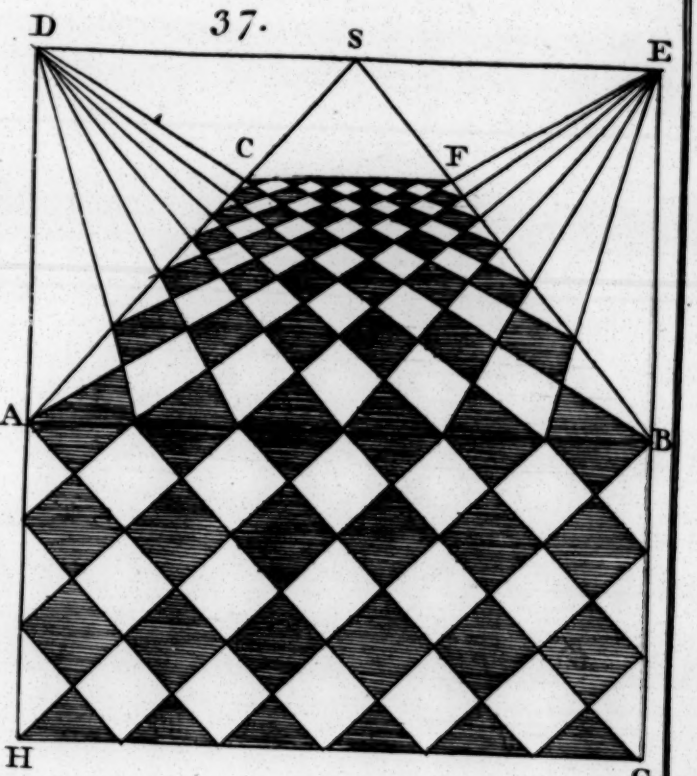
35.



36.



37.



Sect. II

For
are te
terſect
And I
grees
Q wil
BG, a
(by P
tend t
GH i
groun
ture a
III).
CF v
P an
nate
ABF
into
P a
wou
as y

To

I
par
tria
Th
the
lin
A
tri
fo
S
of
A
p

For the figures in the geometrical ſquare ABGH, are terminated by continued right lines which interſect the ground line in an angle of 60 degrees. And IP and IQ alſo making an angle of 60 degrees with the horizontal line DE; therefore P and Q will be the accidental points of the lines AH, BG, and all others parallel to them; conſequently (by Prop. IV.) the appearances of all theſe lines tend to the points P and Q. And ſuch lines as GH in the geometrical plane, being parallel to the ground line AB, their repreſentations in the picture as CF will be parallel to AB (by Cor. 1. Prop. III). Alſo (by Cor. 2. Prop. V.) the diviſions of CF will be equal; and therefore lines drawn from P and Q, thro' theſe equal diviſions, will terminate the remaining triangles, and fill up the ſquare ABFC. And if CF was continued and divided into the ſame equal parts, and lines drawn from P and Q thro' the ſeveral diviſions, theſe lines would form more triangles, and ſo as much ſpace as you pleaſe may be filled with theſe triangles. Fig. 38.

Ex. 7.

To repreſent in perſpective a floor of regular hexagons.

Divide the ground line AB into as many equal parts as you will, and deſcribe a floor of equilateral triangles, as in the laſt example, all in obſcure lines. Then join every 6 triangles together, meeting in the center with all the angular points, and draw lines round them, which will form a hexagon. ABGH is part of the floor drawn upon the geometrical plane. ABCF is the perſpective ſquare, formed by drawing AS, BS to the point of ſight S; and then drawing AE and BD, to the points of diſtance D and E, to cut AS, BS in C and F. All the ſides of the hexagons tend either to the point P or Q; or elſe are parallel to the ground line AB; 39.

Fig. 39. AB; for each of them is the ſide of ſome equilateral triangle, all which is evident by the figure. When the hexagons are all finiſhed in the picture, the ſuperfluous lines may be rubbed out.

PROB. IX.

To draw any given curve line in perſpective, which is ſituated in the geometrical plane.

Find the images of a ſufficient number of points thereof, (by Prob. VI.) and then join them neatly by hand; that is, by drawing a curve line regularly thro' them, making no angles; and the more points are aſſumed, the more exact the work will be.

Ex. 1.

40. Let CGK be any curve, DE the horizontal line, S the point of ſight, D, E the points of diſtance, AB the ground line. Take the points C, F, G, H, I, Z, K of the curve, and let fall the perpendiculars CL, FM, GN, HO, KIP, ZQ. Make L1 = LC, and draw 1D and LS to interſect in *c* the image of C. Make MP = MF, and draw PD and MS to interſect in *f*, the image of F. Make NP = NG, and draw NS and PD to interſect in *g*, the image of G. In like manner make OP = OH, PB = PK, PQ = PI, and QB = QZ. And draw PD, OS interſecting in *h*; and draw BD, QD interſecting PS in *k* and *i*, and laſtly draw QS interſecting BD in *z*. Then *cfghizk* is the repreſentation of the given curve.

Ex. 2.

41. Let FNO be a circle, AB the ground line, DC the horizontal line, S the point of ſight. Draw SE perpendicular to DC, and equal to the principal ray. Draw any number of parallels as FNG thro' the

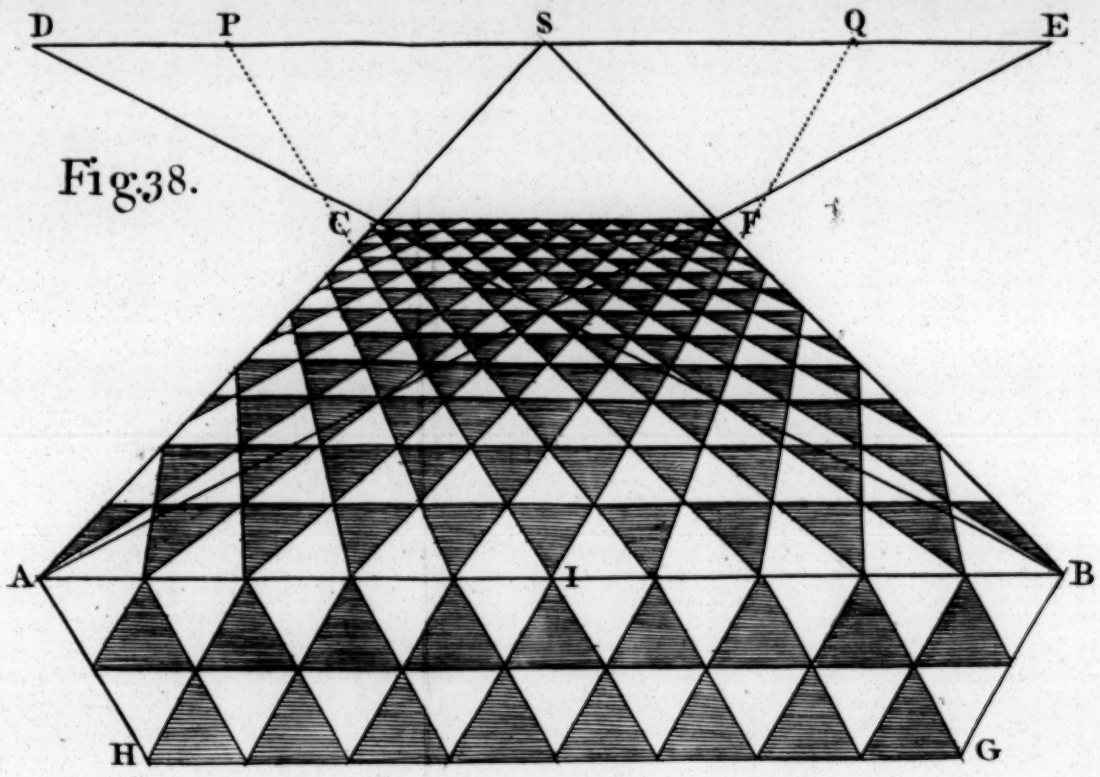
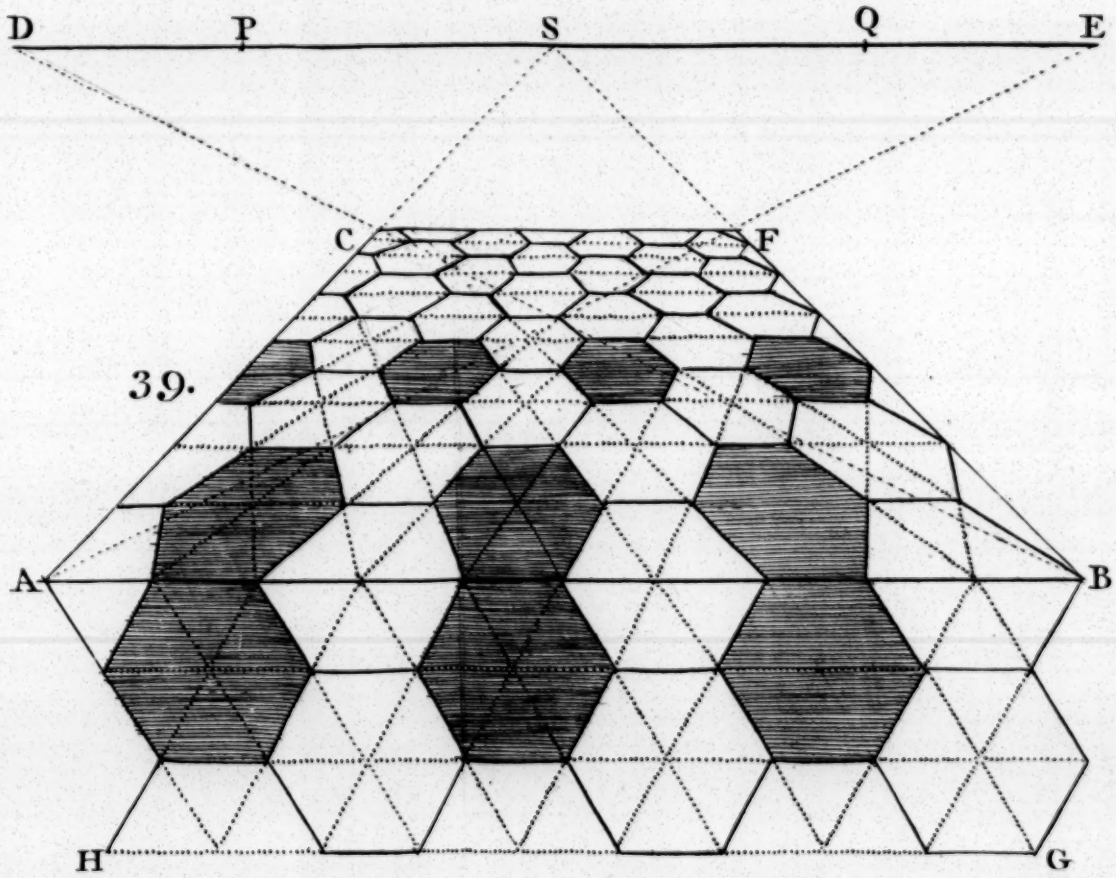


Fig. 38.



39.

the circle to cut the ground line, as at G, and from E draw EC parallel thereto, cutting the horizontal line in C. Then to find the image of any point F, draw EF, and CG interſecting one another in *f*, which is the image of F. In like manner EN being drawn will interſect CG in *n*, the image of N. And thus EO will interſect CI in O, which is the image of O. And ſo of the reſt. Fig. 41.

For ſince EC is parallel to FG, C is the accidental point of FG, and all its parallels. Therefore (Prop. IV.) the images of all theſe parallels tend to C; therefore (by Prob. VI. Method 5); the image of F will be found at *f*, at the interſection of EF and CG. And the like for the reſt of the points.

Otherwiſe for a Circle.

Let DS be the horizontal line, S the point of ſight, D one point of diſtance, AB the ground line. FKH a circle, C the center. Draw the diameter FH perpendicular to AB, and as many lines as you will parallel to FH, as GL, &c. Then find the image of the diameter FH, which is *fh*; and of all the points G, L, K, &c. as *g*, *l*, *k*, as uſual; and thro' *l*, *k*, *g*, &c. draw lines parallel to AB, and continued to 1, 2, 3, ſo that the diameter *hf*, may biſſect the lines *l*1, *k*2, *g*3, &c. Then will 1, 2, 3, &c. be in the curve, which muſt be drawn by hand thro' all theſe points. 42.

SCHOLIUM.

The appearance of a circle will ſometimes be a circle, when it is ſeen foreright; and to find when this will happen, make the height of the eye a mean proportional, between the two parts of the line of ſtation, taken upon the geometrical plane from the eye's perpendicular, to the neareſt and furtheſt points of the original circle. For let AB be the geometrical plane, LB the diameter of the

Fig. 43. circle ſeen foreright, E the eye, EA the eye's perpendicular, ES the principal ray, SH the picture, and FG the image of the circle LB; and that this may be a circle, the cone ELB muſt be cut in ſubcontrary poſition, by the planes AB and SH; that is, the angle ABE muſt be equal to the angle EGF. Draw CL parallel to SH. Then the triangles ABE and AEL are ſimilar; for the angle ABE = EGF = AEL, and the angle at A is common; therefore AB : AE :: AE : AL, and AE is a mean proportional between AB and AL.

PROB. X.

To draw any figure in perſpective, by help of a floor of ſquares; called the Method of Reticulation.

44. Let AB be the ground line, DE the horizontal line, D and E the points of diſtance, S the point of ſight. Make the ſquare ABGH upon the ground line AB, in the geometrical plane, and divide the ſide AB into as many equal parts as you pleaſe, and ſet off the ſame on BG; thro' theſe diviſions draw lines parallel to the ſides of the great ſquare, which will divide it into little ſquares or cells. Then deſcribe the given figure or figures in their true poſition, in the great ſquare ABGH. This done, draw the ſquare ABGH in perſpective, with all the ſmall ſquares within it, by Ex. 4. Prob. VIII. Then deſcribe each part of the object in ſuch a ſquare or cell of the picture, as you ſee it deſcribed in the correſpondent ſmall ſquare in the geometrical plane; and draw the lines in a like ſituation in one as you find them in the other. And thus by going thro' all the parts ſucceſſively, the whole object or objects will be deſcribed. This method by network is very uſeful, for putting any irregular plane figures in perſpective, conſiſting of many parts; which

which would require a deal of time, and the drawing of a great many lines, the common way. Fig. 44.

Examp.

Let IKLMN be a piece of fortification. The part I is ſituated between the ſecond and third ſquare from B, in the firſt croſs row, therefore its image *i* muſt be ſituated between the ſecond and third ſquare of the picture in the firſt croſs row. The part K lies on the ſide of the third ſquare from AB, and of the ſecond from AH; and *k* its image likewiſe lies on the ſide of the third ſquare from AB, and of the ſecond from AC. L lies in the third cell from AH, and ſecond from HG. And its correſpondent *l* lies in the third cell from AC, and the ſecond from CF. M runs into the corner ſquare at G; and *m* runs into the corner ſquare at F. N joins upon the ſide BG, in the third ſquare from B; and *n* joins upon the ſide BF, in the third ſquare from B. And the like for all other parts of it, as the center, &c.

PROB. XI.

To find the image of a right line perpendicular to the geometrical plane, at a given point.

1. Let AB be the ground line, DE the horizontal line, S the point of ſight, D and E the points of diſtance. And let P be the image of the given point. Thro' the point P, draw any line KI, cutting the horizontal line in K, and the ground line in I; make IH = the length of the perpendicular given, and draw HK. Draw PQ parallel to the ground line AB, cutting KH in Q; draw PG perpendicular to AB and equal to PQ; then PG is the repreſentation of the given perpendicular. Fig. 45.

For (by Cor. 2. Prop. III.) the image GP will be perpendicular to AB. Thro' P draw ML per-

Fig. 45. pendicular to AB, then by Prop. XV. as length of the perpendicular HI : to length of its image : : height of the eye ML : diſtance MP : : (ſimilar triangles) KI : KP : : HI : QP; therefore QP, or its equal PG, is the image of that perpendicular.

2. *Or thus.*

Let RV be the diſtance of the perpendicular from the picture, make VO = VR, and VN = given length of the perpendicular; and draw VS, NS to the point of ſight, alſo draw ND to the oppoſite point of diſtance, to cut SV in 1; and drawing 12 parallel to AB cutting NS in 2, then 12 ſet from P in the perpendicular PM to G; gives PG, for the image of the given perpendicular.

For it is evident by the proceſs, that 1 is the image of R, or of the foot of the perpendicular in the geometrical plane. And therefore 1 and P are equidiſtant from AB. And ſince VN = HI, therefore 12 = QP = PG.

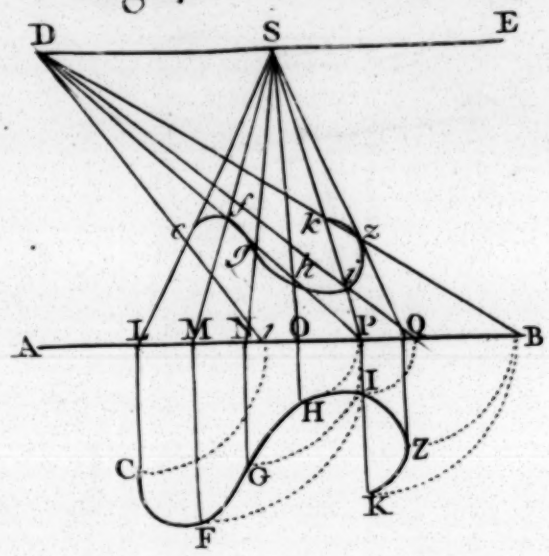
3. *Or thus.*

Draw EB perpendicular to AB, from any point E; in which ſet off BT equal to the given length of the perpendicular. And from any point C in the horizontal line DE, draw CB, CT. From P the image of the foot of the perpendicular, draw PX parallel to AB, and XZ parallel to BT; and laſtly ZG parallel to AB; and PG, in the perpendicular PM, is the image of the given perpendicular.

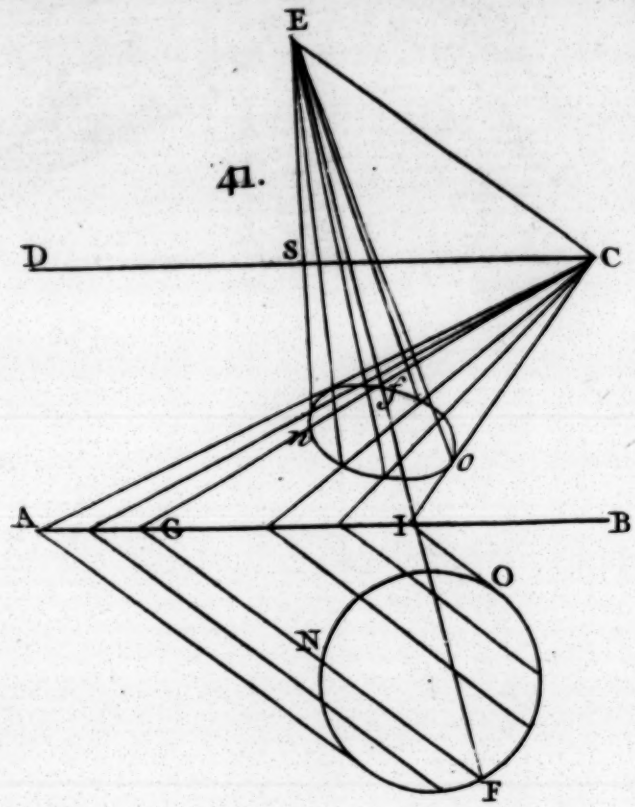
For HI : QP : : KI : KP : : CB : CX : : BT or HI : XZ. Therefore XZ = EP as required.

Cor. *Hence the image of a right line ſtanding obliquely upon the geometrical plain may be found; by letting fall a perpendicular from the top, and finding its*

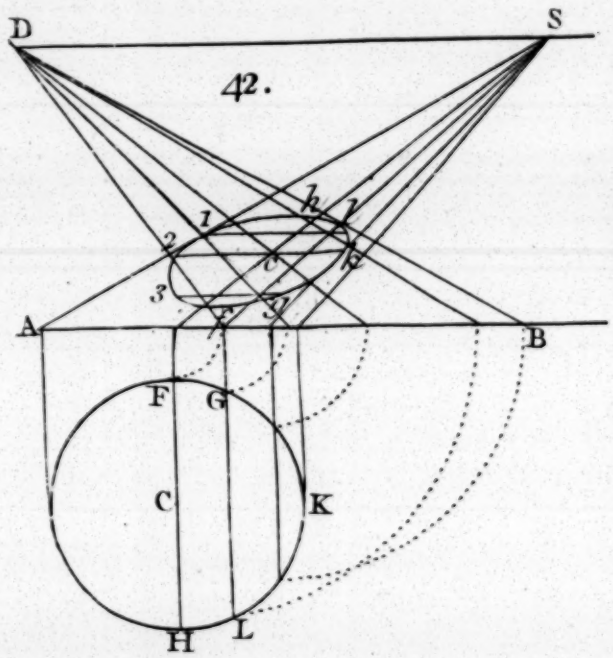
Fig. 40.

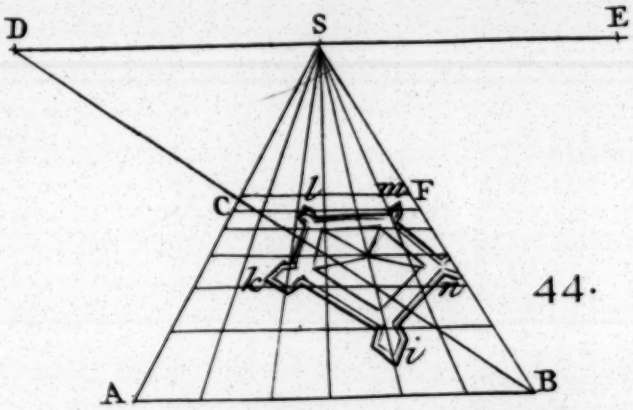


41.

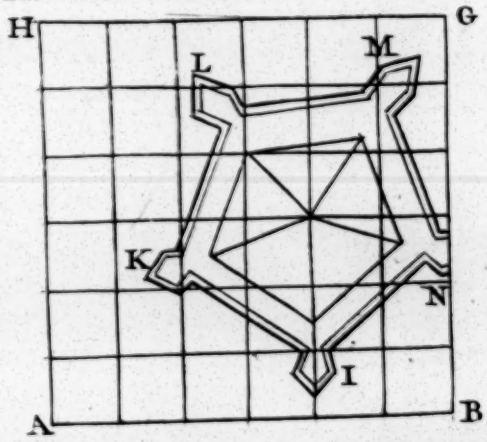


42.

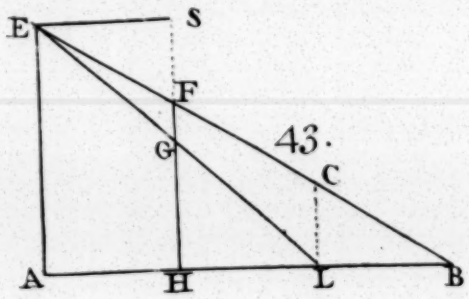




44.

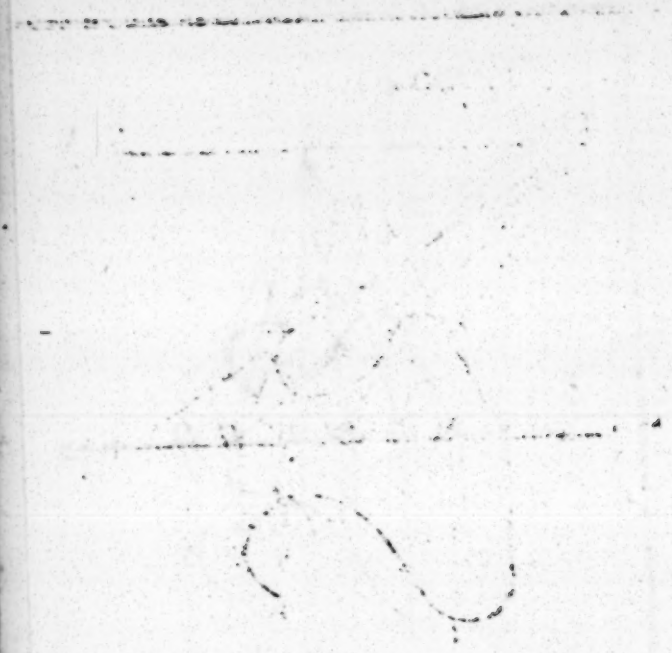


43.



Pl. VI. pa. 52.

Perspective



its appearance, and alſo the appearance of the foot of the line itſelf. Fig. 45.

PROB. XII.

To make a ſcale to ſet off the repreſentations of all equal altitudes.

Let AB be the ground line, DE the horizontal line. Draw BE perpendicular to AB, and ſet off BT equal to the original height on the geometrical plane; and from any point V, in the horizontal line, draw VB, VT. And from any point F required, draw FG parallel to BT, and FG is the apparent altitude. And thus all lines parallel to BT as FG, *fg*, &c. will repreſent equal altitudes at the points F, *f*, &c. 46.

Or if the altitude be greater than the height of the eye; in the perpendicular AD, ſet off AH equal to that altitude; and draw lines from A and H to any point V in the horizontal line DE, as AV, HV. Then any line FG drawn parallel to AH, and comprehended between the lines AV, HV, will repreſent the altitude AH at the point F of the picture.

And if the perpendicular object be on this ſide of the picture, produce the lines VA, VH; and at any point *f* draw *fg* parallel to AH, and *fg* will be the repreſentation of AH at the point *f*.

The truth of this appears by the 3d method in the laſt Prob. This is called a *Flying ſcale*.

SCHOLIUM.

It may be proper ſometimes in finding an altitude, to do it in a ſeparate paper, and afterwards to put it into its proper place in the draught. And this is done where there is a multitude of lines, which will be apt to confound one another.

Fig.

PROB. XIII.

To draw a ſolid body in Perſpective.

From all the angles of the body, let fall perpendiculars upon the geometrical plane, and find the appearance in the picture of all the points where the perpendiculars fall (by Prob. VI); upon all theſe points raiſe perpendiculars (by Prob. XI. or XII.) of due lengths, as taken from the ſolid; the tops of theſe being joined with right lines, as they appear in the body, will give the perſpective draught thereof. But ſuch lines muſt be left out as are hid from the eye behind the body. And this being properly ſhadowed, makes the draught complete, and thoſe planes muſt be ſhaded the deepeſt that are moſt oblique to the eye.

If there be any curve lines in the body, perpendiculars muſt be let fall from ſeveral of their points, on the geometrical plane, and the points of interſection put in perſpective, and perpendiculars erected of a due length; then all the reſt is to be done by hand.

Before you begin, you muſt make choice of ſuch a poſition of the body, that the work may be the eaſieſt done; and that the moſt parts of it may be ſeen, and that it may appear to the beſt advantage. If a parallelopipedon be propoſed, it may be placed with one ſide parallel to the picture, ſo that two faces beſide the top may be ſeen. If a ſolid has a triangle for its baſe, it may be ſet to advantage, by placing one ſide parallel to the picture. And if they be ſet to touch the table it will ſhorten the work.

In drawing all bodies, you muſt have its plan drawn out upon the geometrical plane. And if the body be much compounded, you muſt have the front

front and profile, or the ſections thereof made by perpendicular planes, which is to direct you in the altitudes of the ſeveral parts, and their places and poſitions. You muſt work by the plan for making the baſe of the body; and by the front and profile for getting the altitudes, to be erected upon the ſeveral parts of that baſe. The profile ſhews theſe heights, and the plan ſhews where they are to ſtand. Fig.

Example 1.

To draw a cube in perſpective. This is ſo ſimple, that it needs neither profile nor plan. Let AB be the ground line, S the point of ſight, D the point of diſtance. Let one ſide of the cube be in the ground line at EF. To the point of ſight S, draw ES, FS. Alſo draw DF to the point of diſtance, to cut ES in H. Draw HG parallel to EF, to cut SF in G. Then EFGH is the baſe, upon which at the points E, F, G, H, are to be raiſed perpendiculars equal to EF. Therefore make EI and FK = to EF. Alſo make GL and HM = HG, then I, K, L, M, are the tops of theſe perpendiculars; then draw IK, KL, LM, MI, and they compleat the cube. 47.

For ſince the ſides of the cube are all equal, and EF, FK, KI, IE are all in the picture, their repreſentations are equal to theſe lines. And (by Prob. XI.) HG is the image of EF; and HG, GL, LM, MH being parallel and equidiſtant from the picture, are all equal in appearance. Therefore EGI is the cube drawn in perſpective.

Ex. 2.

To draw a triangular priſm, whoſe baſe is an equilateral triangle. Let one ſide be in the ground line AB, at NO. Find the appearance of the other angle of the baſe at P, by Prob. VI. Then NOP is the baſe. Make NQ equal to the height of the 47.

Fig. 47. priſm, and draw QS, and thro' P draw RV parallel to NQ, then ſet off P*p* equal to RV, and perpendicular to BA; alſo make N*n*, O*o* equal to NQ, and perpendicular to AB. Then draw *no*, *op*, *pn*; and NOP*pno* is the priſm required.

Ex. 3.

48. To draw a parallelopidedon, whoſe baſe is a rectangle. Here P is the plan and Q the profile; AB the ground line, DS the horizontal line, S the point of ſight, D the point of diſtance. By Prob. VI, find the appearance in the picture of all the points in the plan, as D, D, D, D, I, I, I, I, make FO equal to the perpendicular height of the body, draw OS, and thro' D, and I, draw the lines KD, LI parallel to AB: at D, D, erect two perpendiculars equal to KD, as D*d*, D*d*. And at I, I, raiſe two perpendiculars equal to LI, as I*i*, I*i*. Then drawing the ſlant lines as in the profile, and alſo the lines *di*, *ii*, *id*, *dd*. You have the parallelopipedon required.

Ex. 4.

49. To draw a cylinder in perſpective. Let AB be the ground line, SD the horizontal line, S the point of Sight, D the point of diſtance, OGHK the baſe of the cylinder on the geometrical plane. Let the diameter OH be perpendicular to AB, and the diameter GK parallel to it; from all the points O, G, H, K, and as many more intermediate points as you will, let fell perpendiculars to the ground line AB; from which points draw lines to the point of ſight S. Make FC, FA = to FO, FH; draw AD, CD to the point of diſtance D, cutting SF in *h* and *o*, ſo will *ho* be the image of HO. In like manner *g* is the image of G, and *gk* drawn parallel to AB, to cut DC in *k* gives *gk* for the image of GK. And as many intermediate points may thus be found as you deſire. Then joining all theſe points by a curve line, gives the baſe of the

the cylinder *ghko*. Make CB the altitude of the cylinder, and draw BD; and from *o*, *k*, *h*, draw parallels to AB to cut BD; and the lengths of theſe parallels ſet perpendicular at the points *o*, *k*, *h*, *g*, gives the altitude of the cylinder in theſe ſeveral points; and as many more intermediate altitudes may be ſet up as you pleaſe, by the ſcale CDB. Then the curve 1234, drawn thro' theſe points, determines the top of the cylinder. If the cylinder be an oblique one, inſtead of raiſing the perpendiculars from *o*, *k*, *h*, *g*, they muſt be raiſed from the points where the perpendiculars from the top cut the plan, and at laſt the oblique lines *o*1, *k*4, *h*3, *g*2 muſt be drawn, and the curve thro' the points 1, 2, 3, 4, as before. Fig. 49.

Ex. 5.

Let a pyramid be put in perſpective with a hexagonal baſe. Let CEFGHI be the plan of the baſe. From all the angles of the baſe let fall perpendiculars upon the ground line AB, and from the points of interſection, draw lines to the point of ſight S. Then ſetting off the diſtances of theſe angles from AB, from theſe points, towards B, draw lines to interſect the former. Thus H, I, C are projected into *h*, *i*, *c*. Then drawing lines from *h*, *i*, *c*, parallel to AB, will by their interſections with the correſpondent lines, find the points *g*, *f*, *e*, which are the projections of E, F, G. Then if P be the point where the perpendicular from the vertex cuts the baſe, find its projection *p*, as you did the reſt. Set off the altitude of the pyramid from 1 (in the perpendicular P1) to 2, and draw 2S; and thro' *p*, draw 3*p* parallel to AB, which ſet from *p* to V, perp. to AB. Then draw lines from the vertex V to all the angles of the baſe, *efghi*; as V*e*, V*f*, V*g*, V*h*, V*i*, V*c*; or rather leaving Fig. 50.

Fig. 50. ing out V*c* and V*e*, which are inviſible, draw the reſt of the lines, and the pyramid is finiſhed.

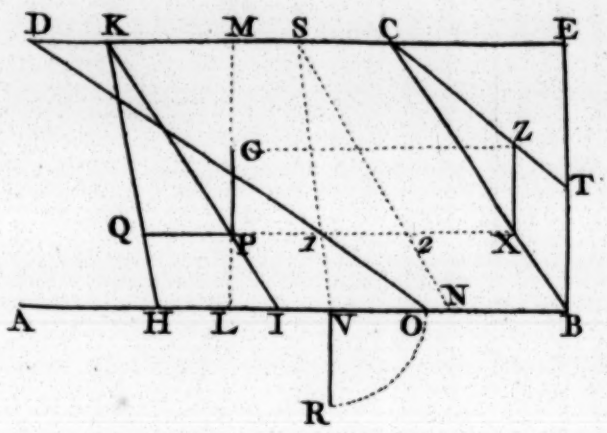
Ex. 6.

51. To draw the fruſtum of a ſquare pyramid in perſpective. Let P be the plan of the baſe, Q the profile. From all the angles of the plan, let fall perpendiculars on the ground line AB, from the points of interſection draw lines to the point of ſight S. Then take ſucceſſively the diſtances of theſe angles from AB, and lay them from their correſpondent perpendiculars towards A; and from the points where they fall, draw lines to the point of diſtance D, to interſect the correſpondent lines drawn towards S; the points of interſection joined by right lines, gives the perſpective of the baſe LMNO, and likewiſe of the interior ſquare 1234. Then make BC the perpendicular altitude of the fruſtum, according to the profile, and draw lines from any point S, as SB, SC. From 1234 draw parallels to AB, to interſect SB, in certain points, from which other parallels muſt be drawn to BC, reaching to SC; and theſe laſt parallels muſt be ſet off from the correſponding points 1, 2, 3, 4, perpendicularly; theſe give the angles at the top at *l*, *m*, *n*, *o*. And the lines *lm*, *mn*, *no* *ol*, being drawn, give the top end of the fruſtum; and laſtly, the lines *l*L, *m*M, and *o*O drawn, and the ſide L*o*, and top *mo*, ſhaded; the fruſtum is finiſhed.

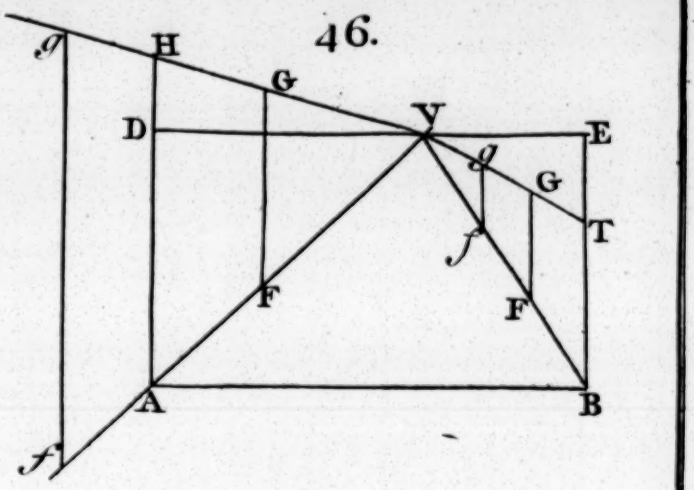
Ex. 7.

52. To repreſent a hollow cube in perſpective. Let P be the baſe, from all the points of it, let fall perpendiculars to the ground line AB, and ſet off their diſtances along BA, as before. Draw lines from the firſt points to S, and from the laſt to D, to interſect in the points of the perſpective baſe, as at L, M, N, O, and 1, 2, 3, 4. Make BC the height,

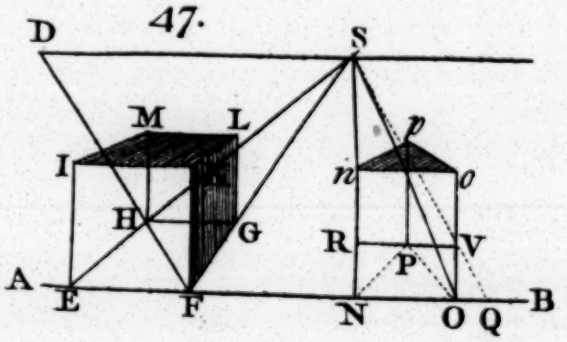
Fig. 45.



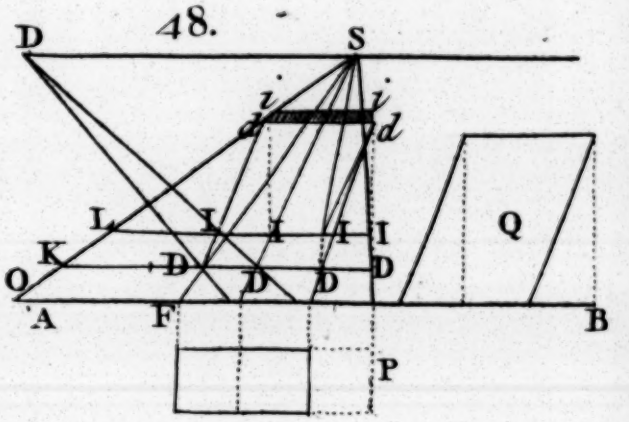
46.



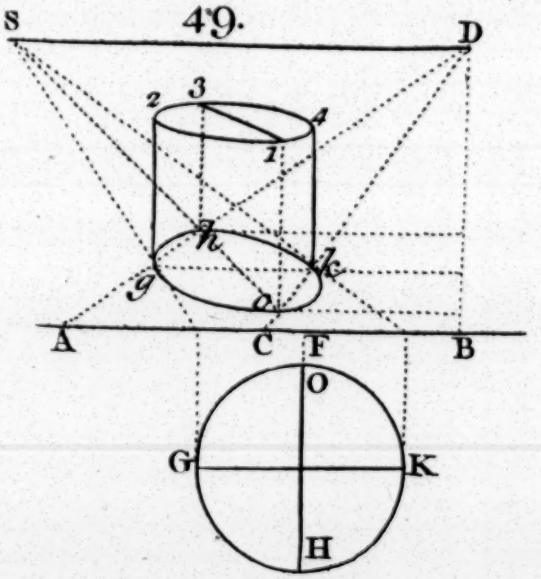
47.



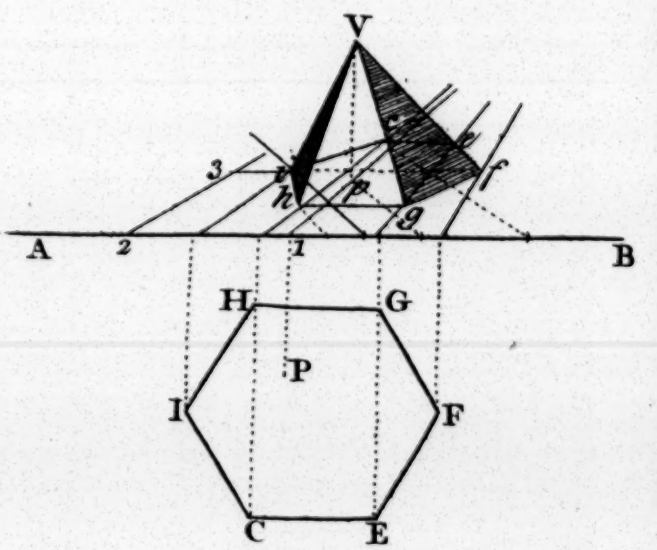
48.



49.



50.



height, equal the ſide of the cube. And make a ſcale SCB as before for ſetting off the altitudes, by drawing from L, 1, 4, O parallels to AB, and from the points of their interſecting SB, draw parallels to CB, for the reſpective altitudes at L, 1, 4, O, and thoſe oppoſite on the other ſide. Several lines are left undrawn to avoid confuſing ſo ſmall a figure; but the way of proceeding is evident. Fig. 52.

Ex. 8.

To draw ſeveral rows of ſquare pillars, at equal diſtances and of equal heights. 53.

Firſt deſcribe the baſes or ſituations of all theſe pillars, which is beſt done by deſcribing a floor of ſquares, as in Ex. 4. Prob. VIII. and upon each row of ſquares taken at equal diſtances, erecting perpendiculars upon the ſeveral baſes, equal to the given height. Here the ſquares are all equal to the deſigned baſes of the pillars; the pillars being all finiſhed, ſuch ſquares as are hid behind the pillars muſt be rubbed out, and the parts that are viſible may be drawn in full.

If inſtead of ſquare pillars you would have cylinders, then the ſquares muſt be made equal to the diameters of the cylinders, and deſcribe the repreſentation of the circles of the baſes within the ſquares, upon which perpendiculars muſt be erected as before. Or if they be cones, perpendiculars muſt be erected from the middle of every correſpondent ſquare, from the top of which two right lines muſt be drawn to the ſides. And after this manner any other priſmatic bodies may be drawn, having any poligonal baſes, deſcribing firſt the baſes of the polygons in their proper ſquares, and erecting perpendiculars at every angle, of the height given; and compleating the ſolids.

As to the ſquare pillars, it is evident that two parallel ſides are parallel to the ground line AB, and

Fig. 53. and the other two ſides, which are alſo parallel, range to the point of ſight S.

Here the foremoſt pillars ſtand in the ground line AB, but if they had been at ſome diſtance, the images of the firſt would have been diminiſhed, which requires ſo much labour, more than if they had ſtood in the ground line.

Ex. 9.

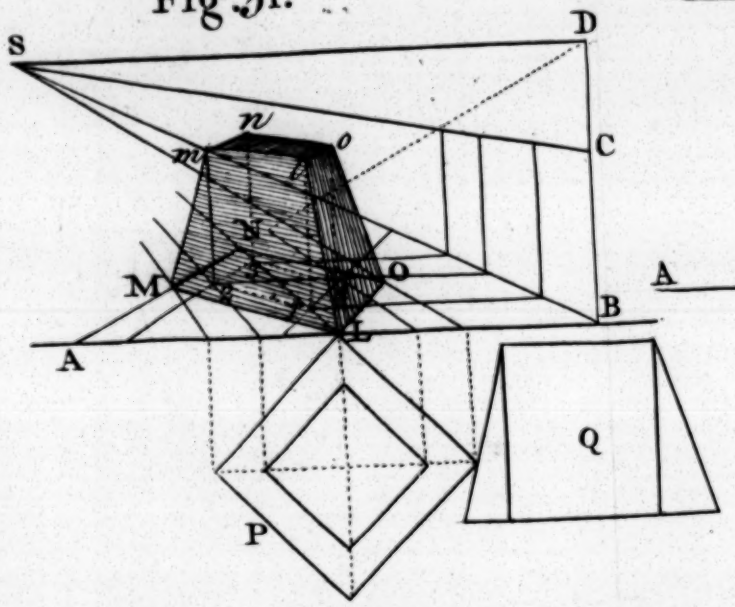
54. To draw an equilateral croſs, ſtanding upright upon the geometrical plane. Draw the plan PP and the profile QQ. Let the plan PP be drawn in perſpective, which will be *cccc*; and from the points *c*, *c*, *d*, *d*, &c. raiſe perpendiculars to the ground line AB, upon which ſet off the heights of the croſs in theſe particular places, according to the profile, but diminiſhed, becauſe it does not ſtand upon the horizontal line. Then all the lines being drawn as in the figure, will give the perſpective appearance of the croſs.

If the croſs had been double, the plan would be the ſame as the profile PQ, and the fore part would come to the ground line AB, and perpendiculars muſt be erected at R, *r*, &c. of a proper height, and lines drawn to terminate the other ends of the croſs.

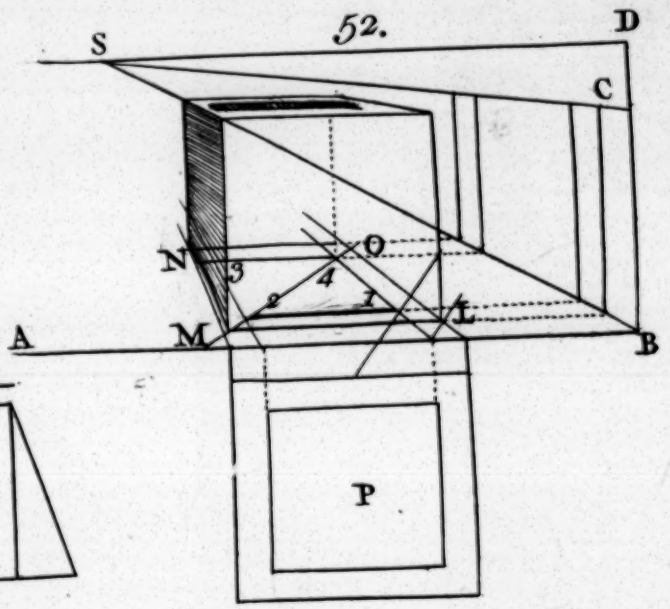
Ex. 10.

55. To repreſent a hollow priſm ſtanding upon an edge. Let P be the plan, Q the profile when cut thro' the middle. Produce the lines of the plan till they cut the ground line AB, and from thence draw lines to the point of ſight S. Then ſetting off the diſtances in the plan, 3, 1, 4, &c. along the ground line towards A, draw lines to the point of diſtance D as uſual, which will find the points 3, 1, 4, &c. on the lines drawn to the point of ſight, correſponding to the points 3, 1, 4 in the plan. Then

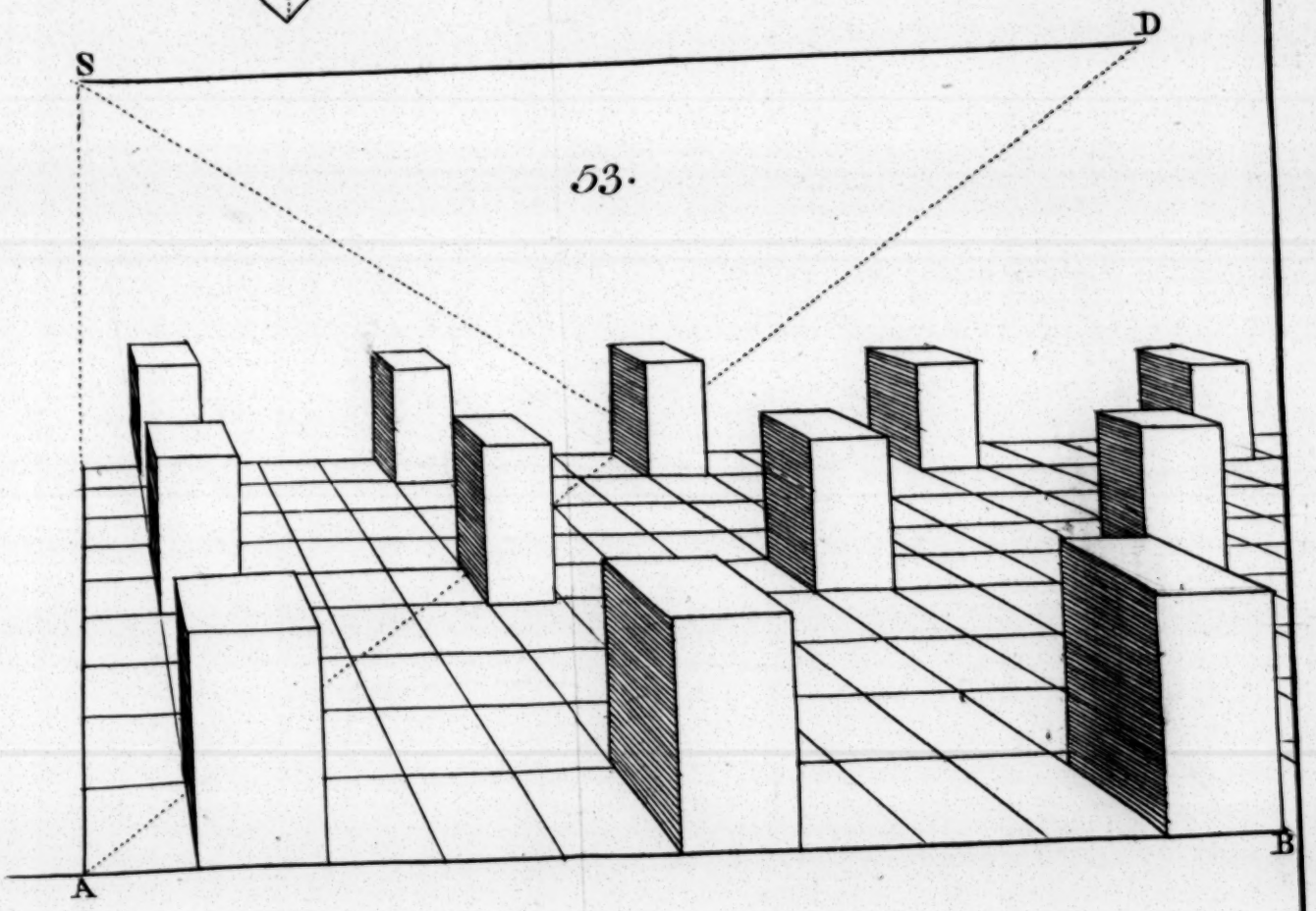
Fig. 51.



52.



53.



Then from the points 2, 2 in the picture, raiſe perpendiculars by the plan, which will give the external angles 2, 2 of the priſm, the points 1 and 1 are the angles of it below, and perpendiculars erected at 1, 1, by the plan, will give the angles 1, 1 at the top. Then the lines 12, 21, 11, 12, 21, 11, being drawn, give the external circumference of the foreſide of the priſm. Then to find the inner circumference bounding the cavity, from 3, 4, &c. in the line S2, raiſe perpendiculars according to the profile, which will be *r*5, *r*5; theſe will give the points 5, 5, 5, 5, being the inner angles of the internal poligon. And from 3, 3 raiſe other perpendiculars, which are equal to *r*2 by the profile, and theſe will give the points 3, 3. Then theſe points connected by right lines will give the inner polygon, 535535. In drawing theſe perpendiculars, they muſt be diminiſhed according to their ſituation by Prop. XV. and ought to be drawn in a ſeparate paper, to avoid ſo great a confuſion of lines; then for the back ſide of the priſm, or that from the eye; ſince the poſition of the priſm is ſuch, that its axis is parallel to the ground line, the interſections of all the faces with one another, will be parallel to the picture; and therefore to find the angles, draw from the points 1, 2, &c. lines parallel to AB, whoſe length muſt be 2B, but diminiſhed according to their ſituations; theſe give the angles 6, 6, 6, which joined by right lines give the appearance of the back ſide of the priſm 666, as far as it is viſible. Then for the inner part, from 3, 5, &c. draw lines parallel to AB as before, but diminiſhed by Prop. XV; which will give the points 7, 7; and lines being drawn from one to another, will give part of the inner poligon on the back ſide, ſo far as it is viſible thro' the hole. And the planes that bound it, being properly ſhaded, we have the repreſentation of the hollow priſm as required.

Fig. 55.

Fig. required. But it is impoſſible to repreſent all the operations in ſo ſmall a draught.

Ex. 11.

56. *To draw a trough in perſpective, being a hollow parallelopipedon.*

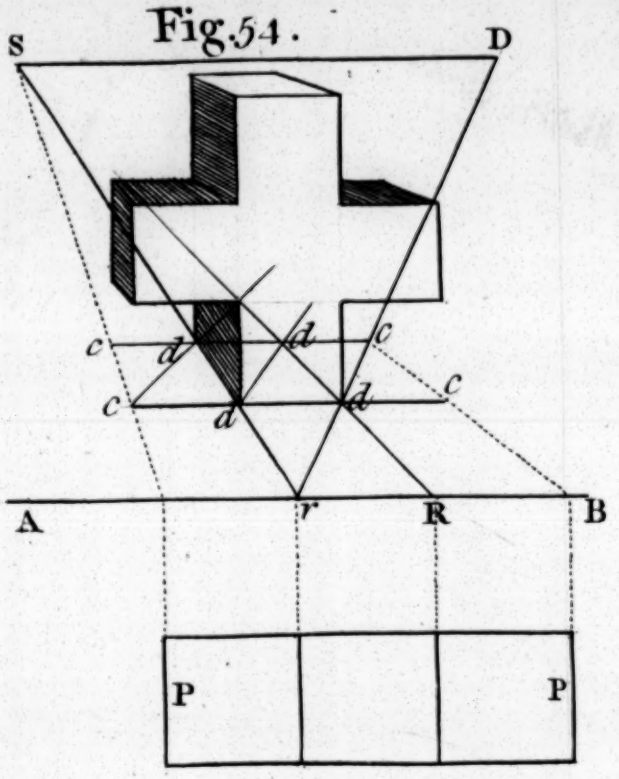
Here P is the plan, and Q the profile or perpendicular ſection. From all the angles in the plan, draw lines perpendicular to the ground line AB, and draw lines from thence to the point of ſight S. Set their diſtances toward A, and draw lines from thence to the point of diſtance D, to cut the former, which will give the points *n*, *n*, &c. in the picture. From the ſeveral points *n*, raiſe the perpendiculars *no*, in the picture, being the repreſentation of *no*, *no*, &c. in the profile. Likewiſe erect the perpendiculars *nr*, *nr*, &c. to repreſent *nr*, *nr* in the profile, which will give the bottom of the trough. Having got the points *n*, *n*, on one ſide, you need only draw *nn*, *nn*, &c. parallel to AB, to cut the lines proceeding from S, which will give the points *n*, *n*, on the oppoſite ſide. In like manner, having got the points *o*, *o*, &c. on one ſide, thoſe on the other ſide are had by drawing parallels to AB, to cut the perpendiculars on the oppoſite ſide, which gives the points *o*, *o*. Then the lines *nn*, *nn*, &c. being drawn give the baſe; and *oo*, *oo*, &c. likewiſe drawn, give the edges at top, on the outer and inner ſide. Likewiſe *rr*, *rr*, &c. drawn makes the bottom. And *no*, *no*, &c. drawn give the corners, which finiſhes the trough.

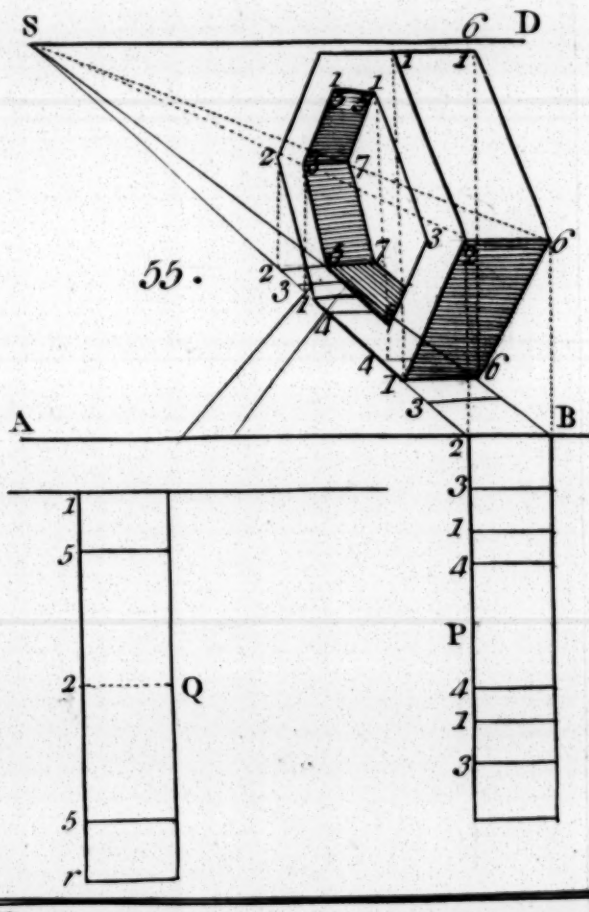
Ex. 12.

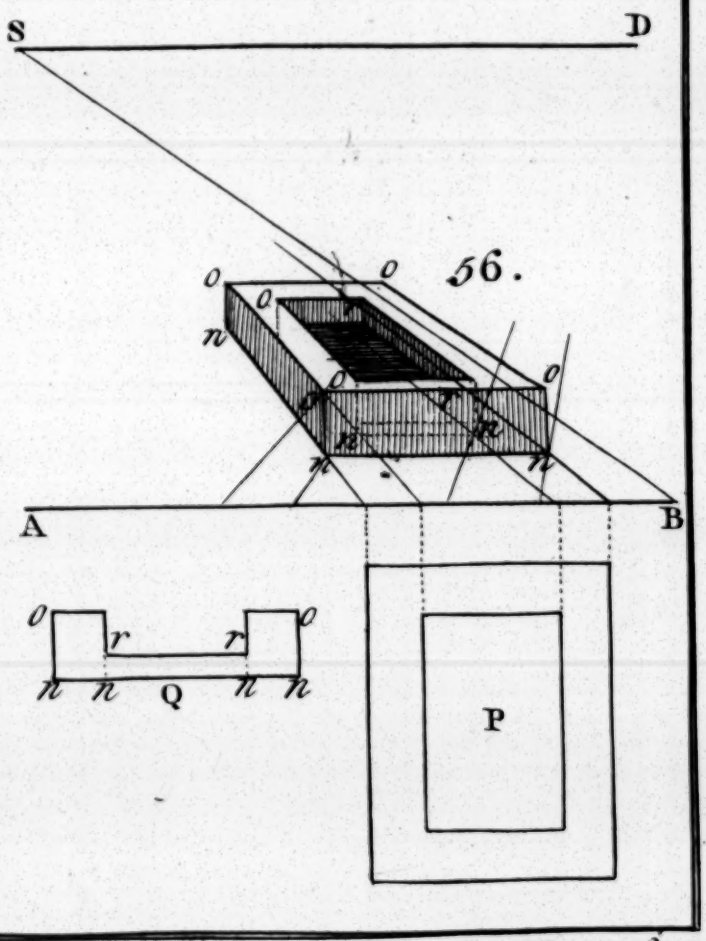
To draw a parallelopipedon leaning againſt a wall.

57. Let P be the plan, Q the profile, or front view. From all the points 1, 2, 1, 1, where the plan cuts the ground line AB, draw lines to the point of ſight S;

Fig. 54.







Perspective

Pl. IX. *pa.* 62.

S; then ſetting the diſtances of 1, 3, &c. towards B, draw lines to the point of diſtance D, to interſect the former in points of the baſe, where perpendiculars are to be erected; which being done, by the profile, diminiſhing thoſe that are on the further ſide; the points 4, 4, 4, and 5, 5, 5, are found, which are the corners at the top. Then thro' theſe points, drawing lines as in the ſolid itſelf, we have 1444 for the firſt face, and 555 for the further face of the ſolid. Draw 45 and 54, and the ſolid is finiſhed. Fig. 57.

PROB. XIV.

To find the image of the Sun in the picture.

Let DR be the horizontal line, AB the ground line, S the point of ſight, D the point of diſtance. Draw QS perpendicular to DR, and make SQ = SD, and make the angle SQN equal to the ſun's declination from the vertical plane, on the left or right, according as the ray drawn thro' the eye and the ſun interſects the picture. Then draw NL perpendicular to DS, and take NR = NQ, and make the angle NRL equal to the ſun's altitude, above or below the horizontal line DS, according as the ſun's ray drawn thro' the eye interſects the picture; then L is the image of the ſun in the picture. Alſo draw LI parallel to DS, to cut QS in I, and N is the point of *declination*, and I the point of *inclination* in the picture. 58.

For if the triangle NQS be raiſed up till it ſtand perpendicular over NS; then ſince the line SQ is equal to SD the diſtance of the eye from the picture; and the angle SQN, is the declination of the ſun from the picture; therefore the ſun's image will be ſomewhere in the line NL. Again, ſince NR is equal to NQ, the diſtance of the eye from N, and the angle NRL is the ſun's altitude, therefore

Fig. 58. fore he is in the point where RL interſects NL, and that is at L.

Cor. *If the ſun is in the plane of the picture, his place* L *will be at an infinite diſtance, in the line* RL.

For then his declination from the vertical plane, is 90 degrees.

PROB. XV.

59. *To find the appearance of the ſhadow of a right line, perpendicular to the geometrical plane, to the picture, or to the vertical plane; caſt upon that plane, or their parallels, by the ſun's light.*

1. Find the appearance of the ſun in the picture at L by the laſt Prob. and alſo the points N and I, the points of declination, and inclination, And let GH be a line perpendicular to the geometrical plane. Thro' the point of declination N and the foot of the line H, draw the line NH, which muſt be terminated by drawing the line LG from the place of the ſun thro' the top of the line G to interſect NH in M, then HM is the image of the ſhadow of GH. Alſo the ſhadow of GH upon the picture or vertical plane will be parallel thereto. And therefore the image will be perpendicular to AB.

For (by Prop. XXI) the ſhadow MH tends to N, the point of declination; and M is the ſhadow of G. Therefore MH is the ſhadow of GH. The reſt follows from Cor. 2. Prop. III.

2. Let FK be a line perpendicular to the picture. Let fall FC perpendicular to the geometrical plane, and thro' N and C draw N*cf*; thro' L the ſun's place and F, draw LF*f*, cutting N*Cf* in *f*, which is the ſhadow of F. Draw *fk* toward S, likewiſe draw LK cutting *fk* in *k*, then *k* is the ſhadow of K, and conſequently *fk* is the image of the

the ſhadow of FK. And by Prop. XXI, *fk* muſt tend towards the point of ſight S. And if the ſhadow of FK falls upon the vertical plane, its ſhadow will be parallel to FK, and its image will tend to S. Fig. 59.

Let PQ be alſo perpendicular to the picture, draw QR parallel to SL, and QR will be the direction of the ſhadow in the picture; ſuppoſing the ſun on your back, and it is terminated by drawing LP from the place of the ſun, to the end P of the line, to cut QR, ſomewhere in the plane ARQ; for the part of the ſhadow that falls without the plane is loſt. For if it cut it at R, then R is the ſhadow of P, and QR the ſhadow of QP in the picture; and if a line perpendicular to the picture was erected at S, it is evident its ſhadow would tend to L. But PQ being parallel to the line at S, its ſhadow by the ſun's light, will be parallel to the ſhadow of the other, or to the line SL, as is evident by Prop. XXI.

3. Let VZ be a line perpendicular to the vertical plane. Thro' the point of inclination I, and the foot of the line Z, draw the line IZ*v*, which is the indefinite ſhadow of VZ, and muſt be terminated by drawing the line LV thro' the place of the ſun, and the end of the line VZ, to interſect the ſhadow Z*v* in *v*; then Z*v* is the ſhadow of VZ in the picture, or its appearance.

For (by Prop. XXI.) the image of the ſhadow *v*Z tends to I the point of inclination, and *v* is the ſhadow of V; therefore Z*v* is the ſhadow of ZV, upon the vertical plane Z*v*I, ſuppoſing the ſun in your face; otherwiſe the ſhadow muſt be directed the contrary way.

Again the ſhadow of VZ upon the geometrical plane will be parallel thereto (by Cor. 1. Prop. XVIII), and therefore parallel to the ground line AB; and this ſhadow being taken as an object, its

Fig. 59. appearance in the picture will alſo be parallel to AB, by Prop. III. Likewiſe the ſhadow of VZ upon the picture will be parallel to VZ, and that ſhadow coincides with its image.

Cor. 1. *Hence if a line be perpendicular to the geometrical plane, the image of its ſhadow upon the vertical plane or the picture, is perpendicular to the ground line. If a line be perpendicular to the picture, the image of its ſhadow upon the geometrical plane and vertical plane, tends to the point of ſight. And if a line be perpendicular to the vertical plane, the image of the ſhadow upon the geometrical plane, or the picture, will be parallel to the ground line. And the ſame of their parallel planes.*

Cor. 2. *When the ſun is in the plane of the picture; the image which a right line perpendicular to the geometrical plane, caſts upon that plane or its parallels, is parallel to the ground line; upon the vertical plane or its parallels, is perpendicular to the ground line.*

For the point N in this caſe is removed to an infinite diſtance along the line SD, and therefore NHM will be parallel to AB, which is the appearance of the ſhadow of GH.

Cor. 3 *When the ſun is in the plane of the picture; the image which a line perpendicular to the picture, caſts upon the picture or its parallels, is parallel to* LS. *Upon the geometrical plane or its parallels, tends to the point of ſight; upon the vertical plane or its parallels, tends to the point of ſight.*

For if a line be erected at L, perpendicular to the picture, LS will be the ſhadow thereof; and all lines parallel to that line, will have their ſhadows parallel to one another and to LS. When the ſhadow falls upon the geometrical or vertical plane; then the real ſhadow will be perpendicular

to

to the picture; and conſequently its image tends to the point of ſight. Fig. 59.

Cor. 4. *When the ſun is in the plane of the picture; the image of a line perpendicular to the vertical plane, falling on it or its parallels, will be perpendicular to the ground line. On the geometrical plane or its parallels, is parallel to the ground line. Upon the picture or its parallels, is an infinite ſurface.*

For the point I being removed to an infinite diſtance, in the line SI, the line IZ*v* will be parallel to IS; or perpendicular to AB. And on the geometrical plane the real ſhadow is parallel to the ground line, and therefore its image is alſo parallel to the ground line.

PROB. XVI.

To find the image of the ſhadow of a right line falling upon an inclined plane, by the ſun's light.

Let CE be a right line or any upright object upon the geometrical plane, PFQ an oblique plane cutting the geometrical plane in the line RV; let OKB be a plane parallel to the vertical plane, and therefore perpendicular to the geometrical plane, and cutting the oblique plane RQ in the line PQ, and the geometrical plane in KB. Let N be the point of declination, and I the point of inclination, L being the ſun's place. Thro' E the foot of the object, and N the point of declination, draw NEG to the foot of the plane OKB. From G raiſe the perpendicular GH, to cut the top of the oblique plane PQ in H, and from F, where the line NG cuts the baſe of the oblique plane, draw FH. Laſtly from the place of the ſun L, draw the top of the object C, the line LCZ cutting FH in Z, where the ſhadow is terminated. Then EFZ is the ſhadow of the object CE. 60.

Fig. 60. For (by Prob. XV.) EFG is the direction of the ſhadow of the object CE, and GH is a part of the ſhadow when the oblique plane is taken away. Therefore FH is the ſhadow upon the oblique plane. And this is terminated at Z by the line LC produced to cut FH in Z; where Z is the ſhadow of C.

Cor. 1. *If the oblique plane* RQ *was taken away, the ſhadow would be* EGH; *the part* EG *upon the horizontal plane, and a part of* GH *upon the perpendicular plane.*

Cor. 2. *If the object* CE *had been higher, a part of the ſhadow would have fallen upon the line* HT; *and then the ſhadow of* CE *would have fallen upon the three planes,* NB, RQ, *and* BO.

Cor. 3. *After this manner it will be eaſy to trace the ſhadow of any perpendicular object thro' any number of planes however ſituated.*

Cor. 4. *If the object is not perpendicular to the geometrical plane, yet its ſhadow may be found, by letting fall a perpendicular from the top as* CE, *and finding the point* Z *as before; and drawing thro' the foot, which ſuppoſe to be at e, the line* NeV, *and then drawing* VZ.

SCHOLIUM.

This ſuppoſes the ſun to be beyond the picture; but if it is behind your back, you muſt raiſe the plane OB perpendicular to the geometrical plane as before, but parallel to the picture. And the reſt of the work will be the ſame. Here if the line NE does not cut the baſe RV of the oblique plane, or if LC does not meet FH; the ſhadow will not fall upon the inclined plane. If a line is drawn from L thro' any point 1 of the line CE, to cut the line NE produced in 2; then will 2FZ repreſent

preſent the ſhadow of the part of the line C1; and E2 the ſhadow of the part E1 upon the geometrical plane. The ſhadow of the point 1 will be repreſented by 2; and ſo the repreſentation of any point of the line CE may be found by drawing a line thro' L and thro' that point; and ſo it will be known whether that ſhadow falls upon the geometrical plane, or upon the oblique plane, or upon the perpendicular one. When the ſun is in the plane of the picture, the points N and I vaniſh, being at an infinite diſtance, and then the line EF muſt be drawn parallel to the ground line AB, till it meet the oblique plane in F, at the common interſection of it with the vertical plane; and proceed with the reſt as before. Fig. 60.

PROB. XVII.

To find the image of the ſhadow of a body upon the geometrical plane, when the ſun is in the plane of the picture.

Let fall perpendiculars from all the angles of the body upon the geometrical plane; from all the points of interſection, draw lines parallel to the ground line, continued on the oppoſite ſide of the light. Then from all the angles of the body, draw lines parallel to the rays of the ſun, to cut the former lines drawn thro' the feet of the perpendiculars, each with each; the points of interſection will determine the ends of the ſhadow. 61.

Examp.

Let DEFG*o* be an inverted pyramid, whoſe vertex is at *o*, in the geometrical plane; from the points D, E, F, G, let fall the perpendiculars D*d*, E*e*, F*f*, G*g*, upon the geometrical plane. From *e*, *f*, *g*, draw the lines *e*1, *f*2, *g*3, parallel to the ground

Fig. 61. line AB. Then thro' E, F, and G, draw the lines E1, F2, G3, parallel to the ſun's ray LS; TSL being the ſun's altitude. The firſt interſects *e*1 at 1, the ſhadow of E; the ſecond interſects *f*2 at 2, the ſhadow of F; and the third interſects *g*3 at 3, which is the ſhadow G. The point D is omitted, being on the other ſide of the body, can give no ſhadow. Then drawing *o*1, 12, 23, *o*3; theſe lines terminate the ſhadow of the pyramid. For 1*o* is the ſhadow of the ſide E*o*, and 12 of the line EF; 23 of the line FG, and 3*o* the ſhadow of the line G*o*. And therefore *o*1230 is the ſhadow of the pyramid *o*DEFG.

PROB. XVIII.

To find the image of the ſhadow of a body upon the geometrical plane; when the ſun is out of the plane of the picture.

Find the ſun's place in the picture, as alſo the point of declination. Then from all the angles of the body let fall perpendiculars upon the horizontal plane. Thro' the places where they fall, draw lines from the point of the ſun's declination, continued beyond the body. Then draw lines from the ſun's place thro' all the angles of the body, to cut the former; the points of interſection will terminate the ſhadow of theſe perpendiculars; and the points of interſection being joined by right lines, terminate the ſhadow of the body, as repreſented in the picture. Note, ſuch lines as lie next the light and caſt no ſhadow may be left out.

Examp.

62. Let CD3 be the fruſtum of a pyramid. Find L the place of the ſun, N the point of declination, in the horizontal line NS; AB is the ground line.

From

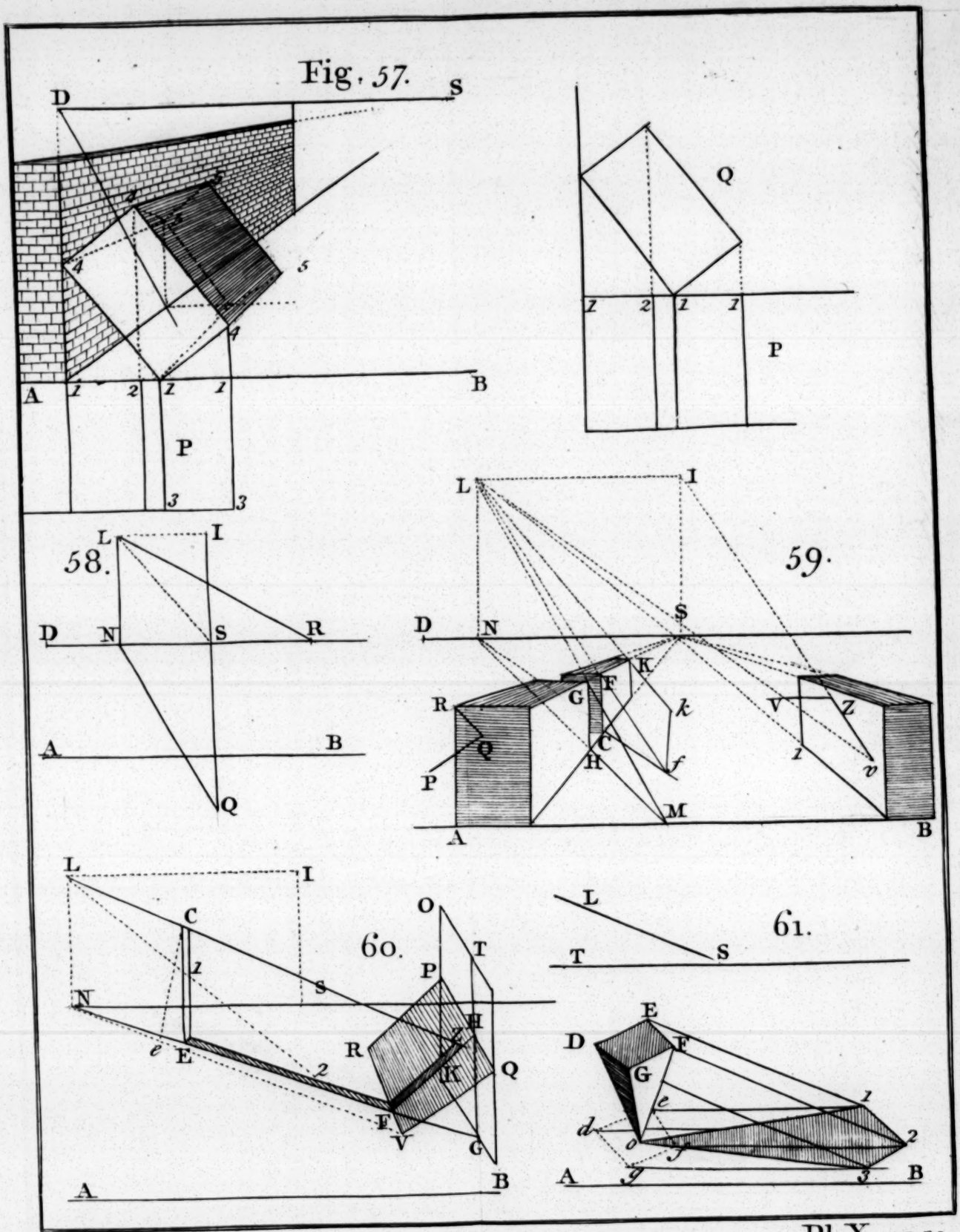


Fig. 57.
D
S
A
B
P
Q
58.
59.
60.
61.
L
I
N
R
Q
K
F
G
H
C
k
f
M
V
Z
v
O
T
S
E
e
d
1
2
3

From the points C, D, E, which lye from the ſun, let fall the perpendiculars C*c*, D*d*, E*e*, upon the horizontal plane. Then thro' *c*, *d*, *e*, draw from N, the lines N*c*, N*d*, N*e*, continued beyond the fruſtum, and theſe are the indefinite ſhadows of the perpendiculars C*c*, D*d*, E*e*. Then to determine the ſhadows of the points C, D, E, draw from L the place of the ſun, thro' theſe points, the lines LC4, LD5, LE6; cutting the former lines drawn from N, in the points, 4, 5, 6, which are the ſhadows of C, D, E. Then drawing the lines 14, 45, 56, 63, they give the ſhadow of the fruſtum required. Fig. 62.

SCHOLIUM.

If part of the ſhadow fall upon an inclined plane, it may be found thereon, by finding the ſhadows of the ſeveral perpendiculars, after the manner of Prob. XVI; and then drawing lines from the extreme points which will terminate the ſhadow. And by Prob. XV. the appearance of the ſhadow of a line falling perpendicular to the picture or vertical plane may be found, or of any number of lines, and conſequently that of a ſurface or ſolid may be found on any of theſe planes.

In drawing any ſorts of ſhadows, they muſt be taken for objects; and therefore their real poſitions, upon the planes they fall on, muſt be known.

PROB. XIX.

To find upon the geometrical plane, the image of the ſhadow of a point, a line, a ſurface or ſolid, expoſed to a ſmall light.

From the point given let fall a perpendicular to the geometrical plane; likewiſe from the light, let fall a perpendicular to the ſame plane, to have the foot of the light. Then from the foot of the light

Fig. draw a line thro' the foot of the perpendicular, which is the direction of the ſhadow. Then draw from the light thro' the given point, a right line cutting the former, then the point of interſection is the ſhadow of the given point in the picture.

And the line contained between the foot of the perpendicular, and this point, is the ſhadow of the perpendicular.

After the ſame manner if another point is given, the image of its ſhadow may be found, and the line drawn between them is the image of the ſhadow of a line drawn between the two given points.

And thus the repreſentations of the ſhadows of all the lines and points in a ſolid may be found, and conſequently the image of the ſhadow of the whole body.

Exam. 1.

63. Let EHG be a cube ſtanding upon the geometrical plane; L a ſmall light, LP a perpendicular upon that plane. Then to find the ſhadow of the point F, draw from the foot of the light P, thro' the foot of the perpendicular G, the line PG3, for the direction of the ſhadow. Then draw thro' the light L and the given point F, the line LF continued to interſect the former in 3; then 3 is the ſhadow of the point F; and G3 is the ſhadow of the line FG.

Again for the point E; from the foot of the light P draw PI thro' the foot of the perpendicular EI, continued; then thro' the light L and the point E, draw LE continued to interſect the other in 2; then 2 is the ſhadow of the point E; and I2 the ſhadow of IE. And if 23 be drawn it is the ſhadow of EF. And G32I is the ſhadow of the plane GFEI. Again, for the point C and line CK; thro' P and K draw the line PK continued; and thro' L and C draw LC to cut the former in

1;

1; then 1 is the ſhadow of C, and K1 of KC. Therefore drawing 12, 23, 3G; we ſhall have 123G for the image of the ſhadow of the cube CGE. Fig. 63.

Ex. 2.

Let DI be a cylinder, CE a perpendicular beam, L a ſmall light, P its foot, GF a perpendicular wall. To find the ſhadow of the beam CE. Thro' the foot of the light P, and the foot of the beam E, draw PEF, which is the direction of the ſhadow upon the geometrical plane; where it touches the foot of the wall at F, raiſe FG perpendicular to the ground line. Then from the light L, and thro' the top of the beam C draw LCG, cutting FG in G, which is the height of the ſhadow. At the place I where the ſhadow meets the cylinder, ſet off the diameter of the cylinder, above the line EF, which will find the higheſt point of the cylinder, thro' which the ſhadow is to paſs, and thro' that deſcribe a ſemicircle for the form of the ſhadow. From L to touch the top of the ſemicircle at I, draw the line LH, which will interſect PF in H, the point to which the ſhadow of the cylinder reaches, at the point I. So that between H and the cylinder, we have the ſhadow of the cylinder; and beyond that is the ſhadow of the beam. As it paſſes over the cylinder, it is in form of a ſemicircle; and ſtreight, from E to the cylinder. The truth of all this appears from Prop. XXII. 64.

Ex. 3.

Let CE be an upright poſt, L a ſmall light, P the foot of it, FG a flight of ſtairs; to find the ſhadow of the poſt CE, upon the ſteps. Thro' P and E, the foot of the light, and the foot of the poſt, draw the line PED, interſecting the horizontal line SD in D; and where it meets with the bottom of the firſt ſtep at 1, there is a plane perpendicular 65.

Fig. 65. pendicular to the geometrical plane, draw the line 12 upon it, perpendicular to the ground line AB; till meeting with another plane at 2, parallel to the geometrical plane; draw 23 towards D upon that plane, till you come to 3, where is another perpendicular plane; upon that draw 34 perpendicular to AB, till you come at 4, where you meet another horizontal plane, thro' which draw 45, ſtill directed to D; and at five where you meet with another perpendicular plane draw 56, ſtill perpendicular to AB; and then upon the next horizontal plane a line from 6 tending to D, which runs off the ſteps, and ſo there is an end of the ſhadow. If from L the point of light, and C the top of the poſt, the line LC be drawn, it cuts none of the former lines, and therefore the remainder of the ſhadow at top is loſt, and no where to be found.

For the plane of the ſhadow of CE, being perpendicular to the horizontal plane, will cut the planes of the ſteps which are alſo perpendicular to the horizon, in the lines 12, 34, 56, which are perpendicular to the ground line AB, which being parallel to one another, and to the picture, their appearances will alſo be perpendicular to the ground line, by Cor. 2. Prop. III. Alſo ſince the plane of the ſhadow interſects the horizontal ſteps in lines parallel to one another and to the line PD, their images 23, 45, all tend to their accidental point (by Prop. IV.), but (Prop. VIII.) their accidental point is in the horizontal line SD, and therefore at D, where PE interſects it.

Scholium.

By a like method, the repreſentation of the ſhadow of any point, line, &c. may be found upon any other plane, by letting fall a perpendicular upon that plane, from the point of the light, and alſo from the point given; as may eaſily be gathered

thered from what is delivered at Prop. XXII. and the foregoing Problems. For the ſhadow of any point in the perpendicular, will be ſomewhere in the direction of the ſhadow, and the direction of the ſhadow is the line drawn upon that plane thro' the foot of the light, and the foot of the line. And the ſhadow of a point in it is in the line drawn thro' the light and that point, and is therefore where it interſects the direction of the ſhadow. And if the line of ſhadow meet any oblique plane, it may be traced upon that plane, by help of another plane drawn perpendicular to the firſt plane, as has been explained in Prob. XVI. in regard to the ſun. Fig.

PROB. XX.

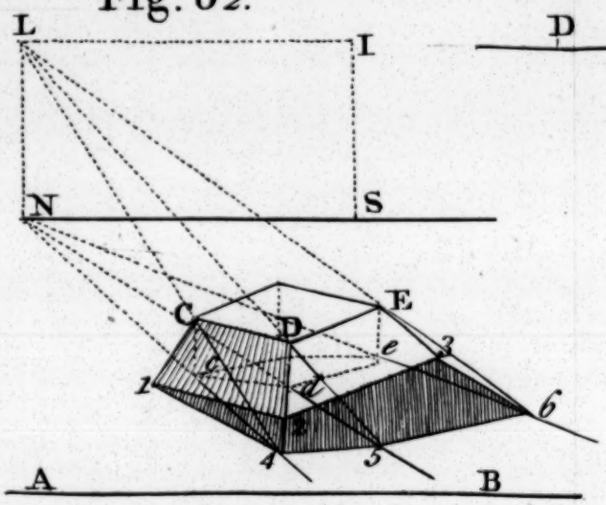
To repreſent ſuch objects as appear by reflexion in the water.

Let AB be the ground line, SD the horizontal line, ABEF a canal or pool of water. GHN a tower with a ſpire upon it. Let LK be the level of the water under the tower HI, make K*h* = KH in the perpendicular H*h*, then the reflexion of the point H by the water, will be at *h*. Alſo let NI be the level of the water juſt under the point G at the vane, make N*g* = NG in the perpendicular G*g*, and *g* will be the reflexion of the point G by the water. And in like manner meaſuring the diſtance from N to the top of the vane at G, and ſetting it down in the line N*g*, gives the reflexion of the top of the vane at 1. If you have a mind to know how much of the tower HN appears in the water, meaſure the diſtance from N or I to the water-ſide at O, and ſet it for I to P; then the part from P to G appears by reflexion, the reſt below being incapable of being ſeen by reflexion, by reaſon of the interpoſed earth between L and O. 66.

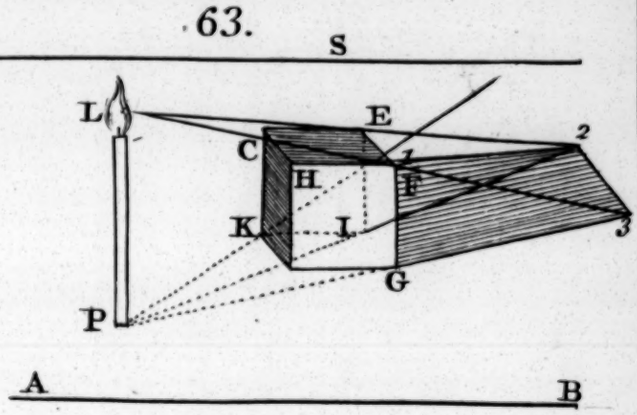
Again

Fig. 66. Again, if T be a tree near the water-ſide, and the level of the water under the top T be at V; then if V2 be made equal to VT, in the perpendicular T2, then 2 will be the appearance of T by reflexion. And thus meaſuring from any part of the tree, perpendicularly to the ſurface of the water or its level, and ſetting that length downwards towards AB, in a perpendicular line, from that ſurface, it will there give the ſame part of the tree as it appears by reflexion. And it will do the ſame for any other object, which is high enough, or near enough the water. For all objects ſeen by reflexion appear to be turned upſide down, the higheſt part appearing the loweſt in the water; and the loweſt to be the higheſt, where they touch at the ſurface. And all this is evident from the known laws of reflexion, which is this, that any object appears by reflexion, to be as far beyond the reflecting plane, as it really is on this ſide; that is, the object and its image are equidiſtant from the reflecting ſurface, and on contrary ſides. Whence if Nl be a part of the reflecting ſurface, then will NG be equal to N*g*. And if LK be the reflecting ſurface, then will KH = K*b*. For the ſame reaſon will TV = V2. As ſome objects may be ſeen directly which cannot be ſeen by reflexion, as the part PK of the tower. So ſome others may be ſeen by reflexion which cannot be ſeen in a direct line. Thus you may ſee the hollow arch of a bridge both by reflexion and a direct view, if your eye is lower than the arch; but if your eye is raiſed higher than the bridge, you may ſee the top of the bridge directly, but not the arch, and yet you may ſee the arch by reflexion; for nothing hinders the rays from the under ſide, from falling on the water, and being reflected to the eye. And thus part of the tree T, which is covered by the other branches,

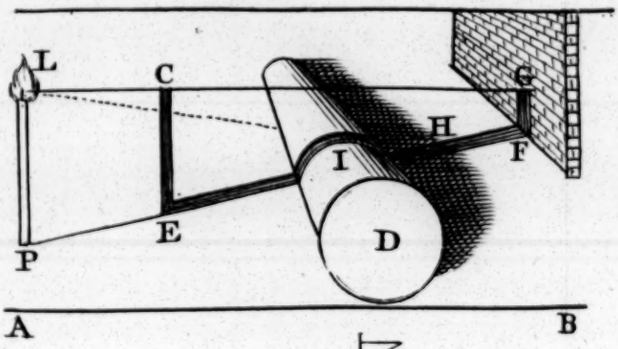
Fig. 62.



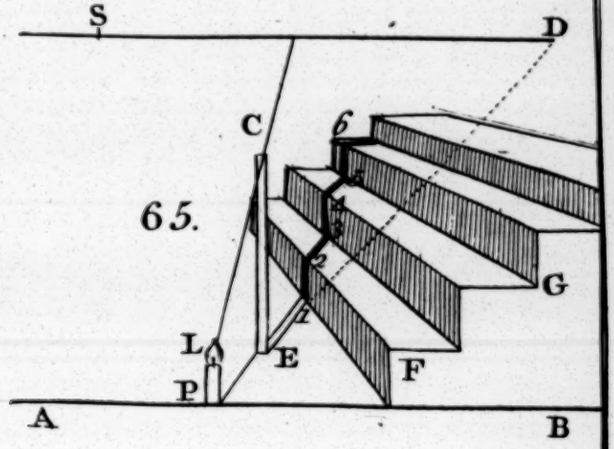
63.



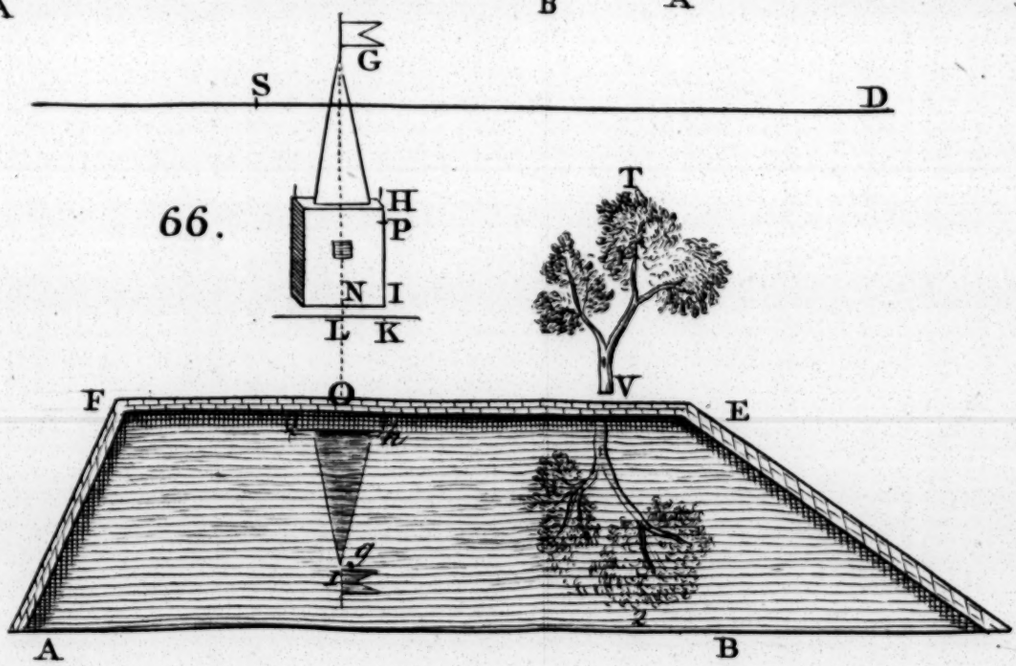
64.



65.



66.



branches, cannot be ſeen directly; but may appear by reflexion in the water. Fig. 66.

PROB. XXI.

To take the perſpective draught of any place, as a Country, City, a Harbour with Ships, &c.

All the problems that have been hitherto delivered, are preparatory to the ſolution of this. For in a general view, all ſorts of objects offer themſelves, of which ſome lye upon the ground, ſome ſtand erect, and ſome ſloaping, ſome are ſtreight, and ſome crooked, bodies ſtanding ſome in one poſition and ſome in another; of various ſorts and forms, ſome near hand, and ſome far off; ſome with ſhadows, and ſome without. So that a perſon before he ſets himſelf to drawing in a general way, muſt make himſelf acquainted with all the foregoing problems, and have them ready in practice, to be applied as there is occaſion. To proceed then,

Take a piece of paper big enough for the purpoſe, as half a ſheet or a ſheet; and near the bottom draw a line for the ground line; and above that, at a diſtance equal to the deſigned height of the eye, draw another line parallel to it, for the horizontal line. The height of the eye, or the diſtance of theſe lines ſhould not be above half the height of the picture, and may be about a third part of it. When the objects are high in the air, the horizontal line muſt be drawn nearer the ground line, to make room for the appearance of the objects above. Then take the point of ſight in the middle of the horizontal line, or in ſome caſes near one end of it; and from it, on one or both ſides, ſet off the points of diſtance, equal to the principal ray, or the diſtance of the eye; which muſt at leaſt be equal to the breadth of the picture; but it ought

Fig. ought to be more, ſo that the picture may ſubtend an angle at the eye, of 45 or 50 degrees. The principal ray, or diſtance from the picture, muſt be more than the height of the eye, it ſhould be half as much more, or ſometimes even twice or thrice as much; and eſpecially when the point of ſight is near one end; and it ſhould never be leſs than eight inches, but may be as much more as you will.

This being done, the diſtances of all the objects, and their heights, muſt be meaſured (or at leaſt eſtimated) by ſome given meaſure, as miles, furlongs, yards, &c. And their ſituations, and poſitions, truly taken; all by the ſame ſort of meaſure. Then they are all to be laid down or planned upon paper, according to their ſeveral places, ſituations and diſtances, to be taken off ſome ſcale of equal parts of a convenient bigneſs, as a ſcale of inches, &c. keeping to the ſame ſcale thro' the whole work. And as the plan or ſituation of all theſe objects is commonly made, by drawing them all below the ground line in an inverted ſituation; yet this will be better done by drawing the plan in a ſeparate paper, of the ſame breadth as the picture, and placing all the objects in their true ſituations. The reaſon of drawing them on the contrary ſide of the ground line, in an inverted ſituation, is to hinder them from interfering with their images in the picture; and to preſerve the true diſtances of all and every part, from the ground line. But this is better done by drawing all the objects above the ground line in their true poſitions in a ſeparate paper; and for that reaſon it is more natural, than inverting all the objects, and ſo changing their places.

The plan of the work being laid down, it muſt be drawn in perſpective in the picture, according to the former rules, one part after another till the whole is inſerted. And if any of the parts be much

much compounded, and requires a great many lines; it will be better to do ſuch part in a ſeparate paper, and then transfer it to the picture. It will be proper to do this as often as there are any difficult parts to be laid down. Fig.

When any buildings, or bodies elevated in the air, happen to be in the view, as few are without; it will often be neceſſary to take the front view and the profile thereof by the ſame meaſure as the reſt; and then lay them down upon paper by the ſame ſcale. And from this draught, they muſt be drawn in perſpective by the rules of drawing ſolids. And here alſo they may be laid down on ſeparate papers when they require many lines; and at laſt transferred to the picture; and thus the picture will be kept clean and neat.

In the practice of all this, one muſt remember that all ſtreight lines are repreſented by ſtreight lines in the picture; and ſuch as are parallel to the ground line, on the earth, will be parallel to the ground line; and thoſe perpendicular to the earth, will be perpendicular to the ground line; and all lines perpendicular to the picture, tend to the point of Sight. Likewiſe all lines parallel to the baſe or the ground, have their accidental points in the horizontal line. And lines parallel to the vertical plane, have their accidental point in the vertical line.

Such objects as ſtand on your right hand, muſt be placed on the right hand of the vertical line; and thoſe on the left, muſt be on the left hand; and thoſe before, in the middle of the picture. And thoſe far off, high up in the picture. And objects that are higher than the eye, muſt be raiſed above the horizontal line.

In deſcribing objects at a great diſtance, their altitudes muſt be diminiſhed in proportion to the diſtance. And therefore a ſcale muſt be made (by Prob. XIII.) for every altitude concerned, which will

ſhew

Fig. ſhew the altitude diminiſhed in every place of the picture. And here the profile muſt be made uſe of, when different heights are concerned. And theſe heights taken from the ſcale, muſt be raiſed upon their proper baſes in the picture, the tops of which joined by right lines, give the boundaries of the bodies, and then theſe bodies may be finiſhed, and duly ſhadowed. When this is done, in reſpect to all the objects that are to appear in the draught, then your view is finiſhed, as far as depends on the rules of geometry and perſpective. When all is done, the ground line, horizontal line, and other lines not belonging to the proſpect muſt be rubbed out; and to make the draught complete, it ought to be coloured, every object with its proper colour.

When objects are ſeen by reflexion in water, they muſt be drawn according to the rules laid down for that purpoſe; and then coloured of the ſame colour as the objects themſelves, but a great deal fainter, by reaſon of the loſs of light, by reflection at the ſurface of the water.

Likewiſe the colours of all objects ought to be ſo much fainter as the diſtance is greater; till at laſt they vaniſh in a blueiſh miſt.

Every one that would be a practitioner in the art of drawing, muſt furniſh himſelf with a drawing board and a Tee. This board or table muſt be right-angled at all the corners, and as big as to contain a ſheet of paper, and made ſmooth and flat. The tee (called ſo from its ſhape) is made after the manner of the carpenters ſquare, with the head thicker than the tong, and muſt be made exactly ſquare. Then a paper being paſted to the board at the four corners, it is ready to draw upon; and when you want to draw parallel or perpendicular lines, apply the head of the tee to one or other of the edges of the board, and then draw by the ſide of the tongue; and thus you may ſoon draw as many

many parallels or perpendiculars as you pleaſe, with- Fig.
out any trouble.

Example 1.

ACEB is a building, whoſe roof is ſupported by 67.
pillars; AB is the ground line, SE the horizontal line, S the point of ſight, and the point of diſtance is far out of the picture. Here the breadth of the houſe is AB, and the height of the eye is SD or EB. The floor here is laid with ſquares, alternately black and white, they lye in rows that run towards the point of ſight, and the other ſides are parallel to the ground line AB. F, F, &c. are two rows of pillars, one on each ſide, theſe are placed at equal diſtances, and range directly towards the point of ſight. G is the further end of the building, being a perpendicular wall of free ſtone; at that end of the houſe are four lamps or tapers H, placed to give light over the room. Between the pillars F, F, and the ſides of the building, are two walks laid with the ſame ſort of ſquares. In the two ſides of the room AC, BC, are two large windows I, I; theſe have each of them three lights, with iron ſtainſhals down the middle, and iron bars acroſs, to keep the jaums faſt. K, K, are arches carried over the building from the top of one pillar to that of another, all manner of ways; upon theſe the roof is built. C, C, &c. is the roof, the two wings on each ſide lye upon the arches L, L, and on others beyond them. The higher part of the roof C, C, ſtands upon the main arches. On the very top at the end of the roof, is placed a vane, to ſhew the wind. Before the entrance is a gravel walk DM, and a roller O to roll it with. On each ſide of it are three rows of trees, which tend to the point of ſight. Along AB is alſo another gravel walk.

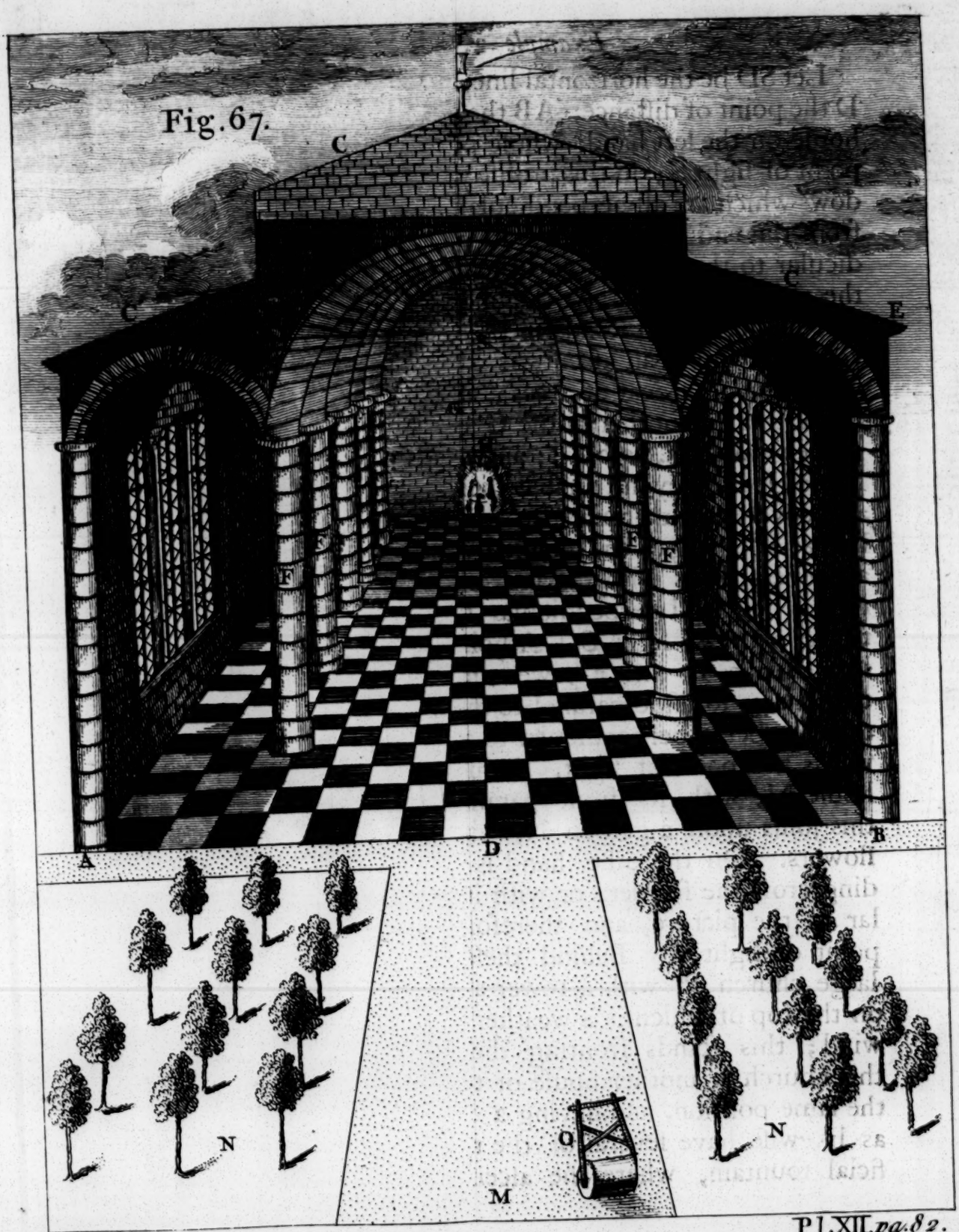
Fig. 68.

Example 2.

Let SD be the horizontal line, S the point of ſight, D the point of diſtance, AB the ground line; C is a houſe on the left hand, whoſe profile runs up to the point of ſight S; in this profile you may ſee the windows which are all perpendicular to the ground line; from the end of it runs a wall E; which is perpendicular to the picture, and therefore it alſo tends to the point of ſight. At the end of the wall is another houſe, which ſtands in the ſame poſition. Beyond at F upon an aſcent, is a houſe ſtanding in a wood among a deal of trees; and at ſome diſtance from this is another houſe G upon a hill, with a garden walled in, before it; and near it is the ſummer houſe H, and beyond theſe is the country, riſing high, and covered with buſhes at I, I. On the ground line at K is a garden incloſed with a wall, in this are ſeveral beds with flowers, and ſome with fruit trees; being all perpendicular to the picture, tend to the point of ſight S. Adjoining to the garden is a church L with a tower indented at the top; this has its foreſide parallel to the ground line; and the ends being perpendicular to the picture, run towards the point of ſight. Behind the church is a houſe M, with its front parallel to the picture; this joins upon a garden N, with trees, herbs and flowers. On the other hand is the houſe O, ſtanding croſs the former, or with its face perpendicular to the picture, and therefore, it tends to the point of ſight S. Behind theſe houſes is another large church P, with a tower and a ſpire upon it, on the top of which is a weather cock, to ſhew the wind; this ſtands fronting the picture. Behind this church is another houſe or two Q, ſtanding in the ſame poſition, and having a chimney ſmoaking, as likewiſe have ſeveral of the reſt. R is an artificial fountain, where the aſcending water plays,

and

Fig. 67.



Pl. XII. *pa. 82.*

Perspective

PERSPECTIVE.

and then falls into the baſon below. T ſeveral inclo-ſures, and ſmall fields. V ſeveral rows of trees. W an obeliſk in a walk between ſome graſs plats. X ſeveral fowls flying in the air. Y a windmill on the top of the hill. Z the clouds hanging in the atmoſphere. Fig.

SCHOLIUM.

From what has been laid down before, concerning the nature of perſpective; the following obſervations may be drawn.

All horizontal planes ſeated above the eye, ſeem to ſink downwards, more and more, the further they are produced; till being infinitely produced, they at laſt become level with the eye. Likewiſe ſuch planes as are below the eye, ſeem continually to riſe upward, the further they go. Alſo thoſe on the right hand approach to the left; and thoſe on the left, towards the right. Therefore floors and pavements of buildings, when continued to a great length, ſeem to a ſpectator to riſe upwards; and the roofs and cielings of buildings, appear gradually to deſcend, and therefore the longer ſuch roofs are, the higher they ſhould be made; to prevent their appearing too near the ground at a great diſtance.

All rows of columns, or of trees; and thoſe of walls or the ſides of buildings, ſeem to the eye of the ſpectator to contract themſelves, the further they run; and continually grow narrower. And therefore the breadth thereof ſhould be greater, where the length is greater; that their appearance to the eye may be the more delightful. And thus the capitals of pillars ſeem to incline downwards, and their pedeſtals to riſe upwards. The horizon ſeems continually to riſe from the eye, and at laſt to appear higher than it really is. But if a row of columns, perpendicular to the horizon, and of equal height, be lower than the eye; the tops of

Fig. them ſeem continually to riſe as well as the bottoms; and thoſe furtheſt off would appear the higheſt. But the contrary happens if they are above the eye, for then the furtheſt would appear the loweſt. Thus the ſurface of the ſea ſeems continually to riſe, and at laſt to be higher than the eye. Hence alſo a plane ſurface continued to a great diſtance will appear hollow. Statues and images raiſed on high, will appear only in their true proportion at a certain height; and at other heights will appear more or leſs deformed or out of proportion. And therefore the optical appearance of any object, in any place, muſt be conſidered.

From the tediouſneſs of drawing a perſpective view throughout, by geometrical rules, as has been now taught; no body ever puts it in practice. But inſtead of that, they make uſe of a mechanical method of doing it, which is this; they take a ſquare box, with a hole in the top facing the object, in which is placed a convex lens; and behind that, within the box is a reflector. This box being placed ſo as to face the country whoſe draught is to be taken, the rays of light flowing from all the objects, enter the convex glaſs, and are reflected downwards by the reflector, which is placed obliquely; and ſo they paint upon a white paper laid at the bottom of the box, the images of all theſe external objects, very diſtinctly if the convex lens be at its true diſtance; if not, the lens muſt be moved in or out, till the images are diſtinct. Then the draughts man, putting his head within the box, has nothing to do, but run over all the out-lines of the ſeveral objects with a pencil; all which being done, they may be finiſhed afterwards. Such a box for taking a perſpective view mechanically you have deſcribed to you in Prob 24. Book IV. of the Optics. For it would be an endleſs taſk to go through all the particulars of ſuch a draught, by the common rules of Perſpective;

ſpective; not only for the number of lines it would require, but for the great length of time requiſite for ſuch a work. And yet theſe rules are the eaſieſt that can be contrived, where the picture is ſuppoſed perpendicular to the horizon. For if the picture was oblique, the labour would be far more, to draw the objects in ſuch a poſition; and new rules are neceſſary to work by, and the objects when drawn make no better appearance than in the common way; and therefore no body would ever follow the practice upon oblique tables, and conſequently they are of no uſe, but merely for curioſity; and to ſhew what can be done by art. Therefore the common method is the moſt uſeful, and its principal uſe, is that of drawing ſingle objects, which require to be done very exactly; and to initiate the drawing by hand, which is by far the moſt uſeful and expeditious method. Hence no landſkips or views of countries are ever drawn by the method of perſpective, but by ſuch mechanical contrivances, as we have ſhewn; and that for the ſake of expedition. And this method is very exact too; for the images are diſtinctly repreſented on the paper, and in their true proportions, as they appear; and the drawing them truly, depends on the ſteady hand of the drawer. Fig.

I muſt obſerve, that when any view or landſkip is to be drawn in a publick place, as in a garden, on the ſcenes in a playhouſe, or the walls of a large room; the point of ſight is not to be taken in the air, where people cannot come. But it ought to be taken where moſt people reſort, and ſtand or ſit to look at it. For then it will appear to the beſt advantage and moſt natural. But if the point of ſight is taken in a different place, the proſpect will appear unnatural and deformed; as may be gathered from Cor. 2. Prop. XII.

Fig.

PROB. XXII.

By the inverſe method of perſpective, having a point given in the picture; to find its original in the geometrical plane.

In what goes before, we have treated only about the direct method of perſpective, or the finding the place of any object in the picture, having its ſituation on the geometrical plane given. We ſhall here add a few propoſitions about the inverſe method, which is finding the original, from having its appearance in the picture. As the place in the picture is found from having that in the geometrical plane, ſo by working backward, its place in the geometrical plane is eaſily found from having that in the picture.

69. Let N be the point given in the picture. From the given point N and the point of ſight, draw the line SE to cut the ground line CB in E. Alſo from the point of diſtance D, thro' the ſame point N, draw the line DC, cutting the ground line in C; then CE is the diſtance of the point, in the geometrical plane, from the ground line. And therefore if E*n* be made perpendicular to AB, and equal to EC, *n* will be the point ſought, but on the contrary ſide of the ground line.

For if *n* was given, its repreſentation at N is found by drawing lines from S to E, and from D to C, to interſect in N, EC being made equal to E*n*. Therefore by the reverſe operation, *n* is truely found.

PROB. XXIII.

69. *Having an object given in the picture, to find its real height, on the geometrical plane.*

Let OP be the object given; find the ſituation F, of the foot O, by the laſt Prob. And thro' F draw

draw FI perpendicular to AB, and from the point of ſight S draw SP thro' the top of it, cutting the perpendicular FI in I. Then FI is the real height of the object upon the geometrical plane. For if FI be the real height, OP will be found to be the apparent height in the picture, by Prob. XI. Fig. 69.

Cor. *In like manner, if* OR *be a line in the geometrical plane, parallel to the ground line; then if* SO, SR *be drawn to cut the ground line in* F *and* G; *then* FG *is the real length of* OR.

PROB. XXIV.

Any oblique line in the picture being given; to find its original in the geometrical plane.

Let OP be the given line in the picture. Find the ſituation of the point P at F, and then its place *p* upon the geometrical plane, by Prob. XXII. Then produce PO to cut the ground line in E, and draw *p*E, which will be the original of PE. 70.

Likewiſe find the original of the point O, which will be at *o*; then *po* will be the original of the line PO, but inverted.

This is evident from the contrary proceſs of finding the repreſentation PO, from having the original *po*.

Cor. 1. *Hence the original of any plane figure given in the picture, may be found. For it is but finding the originals of all the lines, that incloſe the figure; and then you'll have the original figure.*

Cor. 2. *And thus a ſolid being given in the picture, its original may be found, but with more labour. For all the baſe lines, and all the perpendiculars will be found as before; and thence all the ſurfaces that incloſe it, and conſequently the whole ſolid.*

Fig. 70. Cor. 3. *Hence the original of any angle may be found, by finding the original lines that form the angle.*

PROB. XXV.

Two or more lines being given in the picture, whoſe originals are perpendicular to it; and having the real length of any ſuch baſe line; to find the points of ſight and diſtance.

71. Let AB be the ground line of the picture, AEG a houſe, and BHM a wall; both the originals being perpendicular to the ground line AB. Produce any two ſides, as AE, IK, BM, HL, which are perpendicular to the ground line in the geometrical plane; till they interſect, as AE and BM, which interſect in S, then S is the point of ſight.

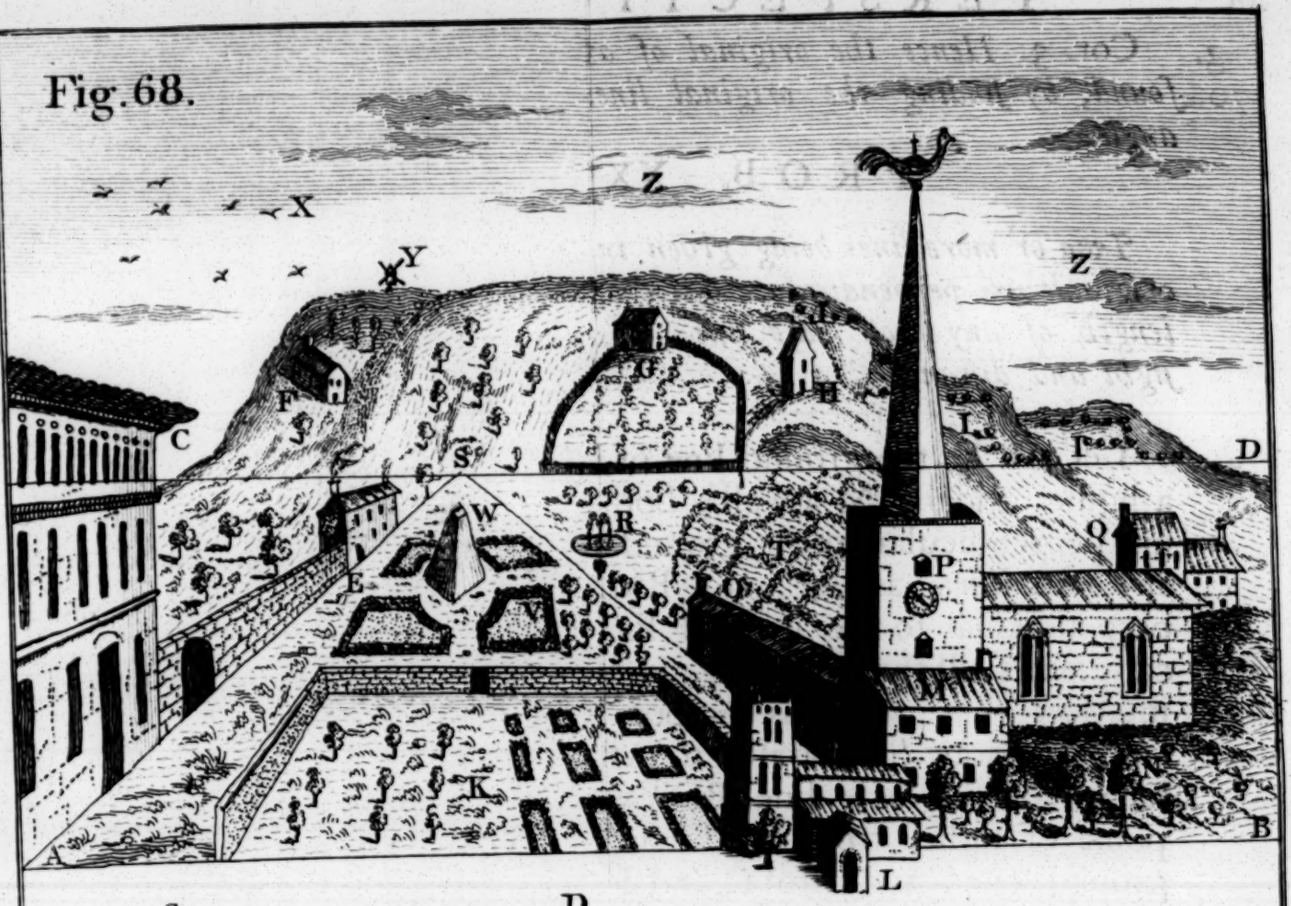
Again draw SD parallel to AB, and ſet the real length of the baſe line BF (which tends to the point of ſight) from its ſituation B to C in the ground line; and thro' C and F draw the line CF cutting SD in D; then is D the point of diſtance; and SD the horizontal line; and the length of SD is the principal ray, or the diſtance of the eye from the picture.

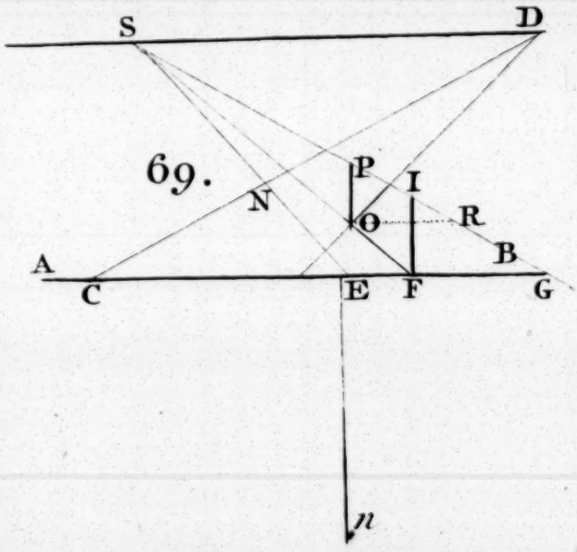
For (by Cor. 2. Prop. IV.) the lines AE and BM tend to the point of ſight at S. And therefore the point of diſtance is ſomewhere in the line SD drawn parallel to AB, which is the horizontal line. But if BC be the diſtance of a point from the ground line, and D the point of diſtance, and DB and DC be drawn to interſect in F, then F will be the image of that object. Therefore on the contrary, when CF is drawn thro' the image at F, it will go thro' the point of diſtance D.

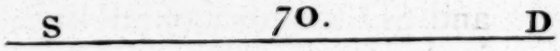
Cor. 1. *Hence, in any perſpective draught, having the horizontal line given; its center, or the point of ſight may be found.*

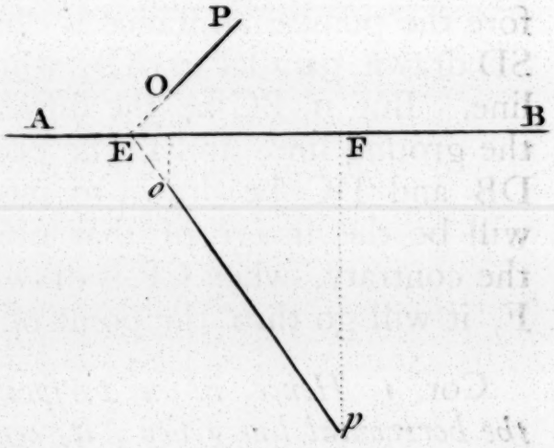
For

Fig. 68.


Z
X
Y
Z
C
F
G
H
I
D
S
W
R
Q
P
E
O
T
K
A
B
L


S
D
69.
P
I
N
O
R
B
A
C
E
F
G
n


S
70.
D


P
O
A
B
E
F
o
p

For produce any line whoſe original is perpendicular to the picture, till it cuts the horizontal line, and there is the point of ſight. Or if there is no horizontal line, produce two ſuch lines till they cut one another. Fig. 71.

Cor. 2. *But the diſtance of the picture, or the principal ray, cannot be found, unleſs the real diſtance of ſome point or object (from the ground line) be known.*

For we only know, that the eye is ſomewhere in the perpendicular, raiſed upon the point of ſight. But if a real diſtance be known, the diſtance of the eye will then be known, as laid down before.

Cor. 3. *If in any draught, it be known that ſeveral original lines are parallel, in the geometrical plane. Then their accidental point, and the horizontal line may be found.*

Produce two of theſe lines till they interſect, and the point of interſection is their accidental point; and that point is in the horizontal line. Therefore thro' that point, draw a parallel to the ground line, and that is the horizontal line.

PROB. XXVI.

Having the angle which any original object ſubtends at the eye, and alſo its image; to find the point of diſtance, or the point of view in the air.

Make the angle CEF upon the horizontal line, equal to the angle which the original object ſubtends at the eye. Let OP be the image of it in the picture. Make CF = OP, and place CF in the ſame poſition to EC, as the image has to the viſual ray paſſing thro' O. Then CE is the diſtance of the eye from the point O in the picture. When that diſtance is had, erect a perpendicular at S the point of ſight (found before), which muſt ſtand Fig. 72.

Fig. 72. ſtand upright upon the plane of the picture; then the diſtance EC ſet from O, will interſect that perpendicular in the point of view. Then the diſtance between the point of view and point of ſight, or the principal ray, ſet from S to D, gives the point of diſtance D.

Having ſaid ſomething before of inclined pictures, I ſhall add a little more here; more out of curioſity, than any real uſe they are of.

PROB. XXVII.

To draw objects in perſpective upon an inclined picture.

I have ſhewn in Prop. XXIII. that the rules for drawing upon inclined pictures are the ſame as thoſe for upright ones; taking here a line parallel to the vertical line, inſtead of the height of the eye, in the upright picture; and taking lines ſtanding on the geometrical plane, parallel to the vertical line, inſtead of perpendiculars. I ſay lines in this poſition, and lines parallel to the ground line, will be drawn by the very ſame rules. Therefore in theſe caſes, you muſt work by Prob. VI. VII. VIII. and Prob. XI. for altitudes, And when perpendiculars are given upon the geometrical plane, you muſt firſt draw lines from their tops parallel to the vertical line, to cut the geometrical plane, and there you have their baſes; and then finding the images of one of theſe baſes, you muſt draw an oblique line obſcure. Then find the image of the foot of the perpendicular itſelf, and joining this laſt image, and the top of the former obſcure line, by a right line, you'll have the image of that perpendicular upon the inclined plane; and ſo of all the reſt of the perpendiculars. By this means planes may be drawn, and ſolids perpendicular upon

on the horizon or geometrical plane. And any Fig.
other ſort of oblique lines may be drawn, by a like method, firſt drawing a ſlant line parallel to the vertical line; then you have two lines meeting at the top, or whoſe top is common to both; find the images of both the baſes or feet, and the altitude of the ſaid ſlant line, then join the top and the proper baſe belonging to that line, ſee Cor. Prop. XXI.

But the beſt way is, firſt to draw the profile of the picture, and the principal lines belonging to it; and find the accidental point of lines perpendicular to the horizon; or of any other lines, which are to be drawn in perſpective. Thus,

Let SH be the picture, E the place of the eye, 73.
or the point of view. Draw ES and AH parallel to the horizon, draw EC perpendicular to AH, to cut SH produced in F, then (by Def. 17.) F is the accidental point of all lines parallel to EC, or perpendicular to the horizon. Alſo draw EA parallel to SH; and then EA is to be taken for the height of the eye, ES the principal ray, AH the line of ſtation, and SH the picture. Then in order to draw upon ſuch a picture, (in Fig. 74.) draw SD for the 74.
horizontal line, and make SD equal to SE, and D is the point of diſtance. Draw SH perpendicular to SD, and equal to SH (fig. 73.) for the vertical line; and draw AHB parallel to SD for the ground line; or this may be drawn firſt, if you will; alſo make HF equal to HF (fig. 73), and F will be the accidental point of all lines perpendicular to the horizon. Thus all the requiſites are transferred from fig. 73. to fig. 74. which done, the operations are to be performed as before-mentioned. Examples will explain the thing.

Ex. 1.

To find the image of a given point P. Let 74.
fall PI perpendicular to AB, and ſet PI from I to A.

Fig. 72. ſtand upright upon the plane of the picture; then the diſtance EC ſet from O, will interſect that perpendicular in the point of view. Then the diſtance between the point of view and point of ſight, or the principal ray, ſet from S to D, gives the point of diſtance D.

Having ſaid ſomething before of inclined pictures, I ſhall add a little more here; more out of curioſity, than any real uſe they are of.

PROB. XXVII.

To draw objects in perſpective upon an inclined picture.

I have ſhewn in Prop. XXIII. that the rules for drawing upon inclined pictures are the ſame as thoſe for upright ones; taking here a line parallel to the vertical line, inſtead of the height of the eye, in the upright picture; and taking lines ſtanding on the geometrical plane, parallel to the vertical line, inſtead of perpendiculars. I ſay lines in this poſition, and lines parallel to the ground line, will be drawn by the very ſame rules. Therefore in theſe caſes, you muſt work by Prob. VI. VII. VIII. and Prob. XI. for altitudes, And when perpendiculars are given upon the geometrical plane, you muſt firſt draw lines from their tops parallel to the vertical line, to cut the geometrical plane, and there you have their baſes; and then finding the images of one of theſe baſes, you muſt draw an oblique line obſcure. Then find the image of the foot of the perpendicular itſelf, and joining this laſt image, and the top of the former obſcure line, by a right line, you'll have the image of that perpendicular upon the inclined plane; and ſo of all the reſt of the perpendiculars. By this means planes may be drawn, and ſolids perpendicular upon

on the horizon or geometrical plane. And any other ſort of oblique lines may be drawn, by a like method, firſt drawing a ſlant line parallel to the vertical line; then you have two lines meeting at the top, or whoſe top is common to both; find the images of both the baſes or feet, and the altitude of the ſaid ſlant line, then join the top and the proper baſe belonging to that line, ſee Cor. Prop. XXI. Fig.

But the beſt way is, firſt to draw the profile of the picture, and the principal lines belonging to it; and find the accidental point of lines perpendicular to the horizon; or of any other lines, which are to be drawn in perſpective. Thus,

Let SH be the picture, E the place of the eye, or the point of view. Draw ES and AH parallel to the horizon, draw EC perpendicular to AH, to cut SH produced in F, then (by Def. 17.) F is the accidental point of all lines parallel to EC, or perpendicular to the horizon. Alſo draw EA parallel to SH; and then EA is to be taken for the height of the eye, ES the principal ray, AH the line of ſtation, and SH the picture. Then in order to draw upon ſuch a picture, (in Fig. 74.) draw SD for the horizontal line, and make SD equal to SE, and D is the point of diſtance. Draw SH perpendicular to SD, and equal to SH (fig. 73.) for the vertical line; and draw AHB parallel to SD for the ground line; or this may be drawn firſt, if you will; alſo make HF equal to HF (fig. 73), and F will be the accidental point of all lines perpendicular to the horizon. Thus all the requiſites are transferred from fig. 73. to fig. 74. which done, the operations are to be performed as before-mentioned. Examples will explain the thing. 73. 74.

Ex. 1.

To find the image of a given point P. Let fall PI perpendicular to AB, and ſet PI from I to A. 74.

Fig. 74. A. Then draw SI from the point of ſight S, and DA from the point of diſtance D; and where they interſect at *p*, is the image of P. This is juſt the ſame proceſs as in the upright picture.

Ex. 2.

To find the image of a right line, parallel to the ground line.

Let PQ be the line given; from P and Q let fall perpendiculars upon the ground line AB, to interſect it at I and K. From I and K draw lines to the point of ſight, IS, KS. Set IP from I to A, and draw AD to the point of diſtance D, to interſect IS in *p*. Draw *pq* parallel to AB, cutting KS in *q*; then *pq* is the image of PQ. Or find the point *q* the repreſentation of Q, as *p* was found, and draw *pq*.

Ex. 3.

To find the image of any line in the geometrical plane.

Let PL be the line given. Find *p* the image of P, one end of the line as in the firſt example. And likewiſe *l* the image of L the other end of the line, the ſame way; then draw *pl*, which will be the repreſentation of PL, as required.

Ex. 4.

To find the image of a plane figure given in the geometrical plane, on an inclined picture. Let PQLR be a ſquare, whoſe two oppoſite ſides are parallel to the ground line AB. Find the repreſentations of all the points P, Q, L, R; at *p*, *q*, *l*, *r*; and draw the lines *pq*, *ql*, *lr*, *rp*; then *pqlr* is the image of the ſquare PQLR.

Or thus, let fall perpendiculars from all the angles upon the ground line AB, to interſect it at I and K; draw IS, KS to the point of ſight S. Set

off

off IR from I towards A, from which point draw a line to the point of diſtance D, interſecting IS in *r*. Alſo ſet off IP from I toward A, and from that point draw a line likewiſe to the point of diſtance D, to interſect IS in *p*. From *r* and *p* draw two lines parallel to AB, as *rl*, *pq*, interſecting KS in *l* and *q*; then *pqlr* is the repreſentation of the ſquare PQLR, whoſe two ſides are parallel to the ground line AB. Fig. 74.

Ex. 5.

To draw the image of a line parallel to the vertical line of the inclined picture. Let SD be the horizontal line, AB the ground line, S the point of ſight, D the point of diſtance, as before; and let P be the foot of the line in the geometrical plane. Draw PI perpendicular to AB, make IL = PI. Then draw SI, LD to interſect in *p* the image of P, the foot of the line. Make IA equal to the length of the line, which ſtands not perpendicular but ſloping; draw AS, and from *p*, draw *p*G parallel to AI, cutting AS in G. Draw *pg* parallel to SH, and equal to *p*G, then *pg* is the image of the line given. 75.

If the line had ſtood in the ground line at I, you had no more to do, but to draw a line from I parallel to SH, and equal to IA, the length of the line propoſed.

Ex. 6.

To find the repreſentation of a line perpendicular to the horizon. Let Q be the foot of the perpendicular upon the geometrical plane, and CN its height. Find the image of Q, by Ex. 1. which will be at *q*. The picture being in the poſition SH, fig. 73. From the top of the line C, draw CM parallel to the vertical line, then MCN will be the profile of the lines CN, CM. Therefore make QO = NM, and O will be the ſituation of

Fig. 75. M upon the geometrical plane. Then find the image of O, which will be at *o*. Alſo find the image of MC whoſe foot is at *o*, by Ex. 5. this will be *oc*, which is drawn parallel to SH, and equal in repreſentation to MC. Laſtly, draw *cq*, and that will be the repreſentation of the perpendicular CN, ſtanding at Q, whoſe image is *q*.

For ſince CM is drawn parallel to the vertical line, the line NM is perpendicular to the ground line or baſe of the picture; therefore the triangle CMN is the profile of the ſlant line, the perpendicular line, and the baſe. And ſince N is at Q, M will be nearer the picture as at O; therefore *q* is the repreſentation of Q or N, and *o* is the repreſentation of O or M; and *oc* being the repreſentation of MC, therefore the triangle *ocq* is the repreſentation of the triangle MCN; and *cq* is the repreſentation of CN.

Otherwiſe.

Find the accidental point F of lines perpendicular to the geometrical plane, as directed before. Alſo find *q* the repreſentation of Q, the foot of the perpendicular as before. Thro' F draw *qc* for the indefinite perpendicular ſtanding at *q*, which muſt be terminated thus; draw the ſlant line MC as before, and ſet its length from K to T in the perpendicular KV; then draw ST which will cut *qc* in *c*, the top of the perpendicular line required.

For the images of all lines perpendicular to the horizon tend to the accidental point F; and the image of N or Q is at *q*. Draw *oc* perpendicular to AB, to cut SK in *o*. Then if KT be the appearance of CM at K in the ground line; *oc* will be its appearance at *o*, by Prob. XI. therefore *c* is the top of the image, and therefore *qc* is the image of the perpendicular required.

Ex.

Fig. 76.

Ex. 7.

To draw the repreſentation of any ſloaping line. Let the foot of the given line be at P in the geometrical plane; ſuppoſe a line drawn from the top of this line parallel to the vertical line, to cut the geometrical plane in the point Q, and let the length of that be CM. Find the repreſentations of the points P, and Q, at *p* and *q*, by Ex. 1. Set off MC from K to B, in the horizontal line AB, and draw SB from the point of ſight. Draw *ql* parallel to AB, to cut SB in *l*; and draw *qc* parallel to SH and equal to *ql*, then *c* is the image of the top of the line. Therefore draw *cp*, and it will be the image of the given ſloaping line drawn to P. For it is evident by the proceſs in Ex. 5, that *cq* is the repreſentation of the line CM, drawn to Q. Therefore *cp* is the image of the given line.

By the help of this example any ſolid may be drawn upon an inclined picture; by drawing all the out lines thereof, and joining their tops by right lines, as thoſe in the ſolid are drawn. But it is beſt to draw all the parts in ſeparate papers.

Ex. 8.

74. To draw an oblique parallelopipedon, whoſe axis is parallel to the vertical line, and one face parallel to the picture, and the baſe a ſquare.

Let AB be the ground line, S the point of ſight, D the point of diſtance. Let the baſe of the ſolid be deſcribed in the geometrical plane, according to the given ſituation, as at PQLR, which is a ſquare baſe. Draw this in perſpective, as ſhewn in Ex. 4. which let be *pqlr*. This done, we muſt proceed to raiſe the heights, at the places *p*, *q*, *l*, *r*, as directed in Ex. 5. thus, ſet the length of the axis or any ſide of the parallelopipedon, from B to C in the perpendicular BC; draw the lines SB, SC; then

SBC

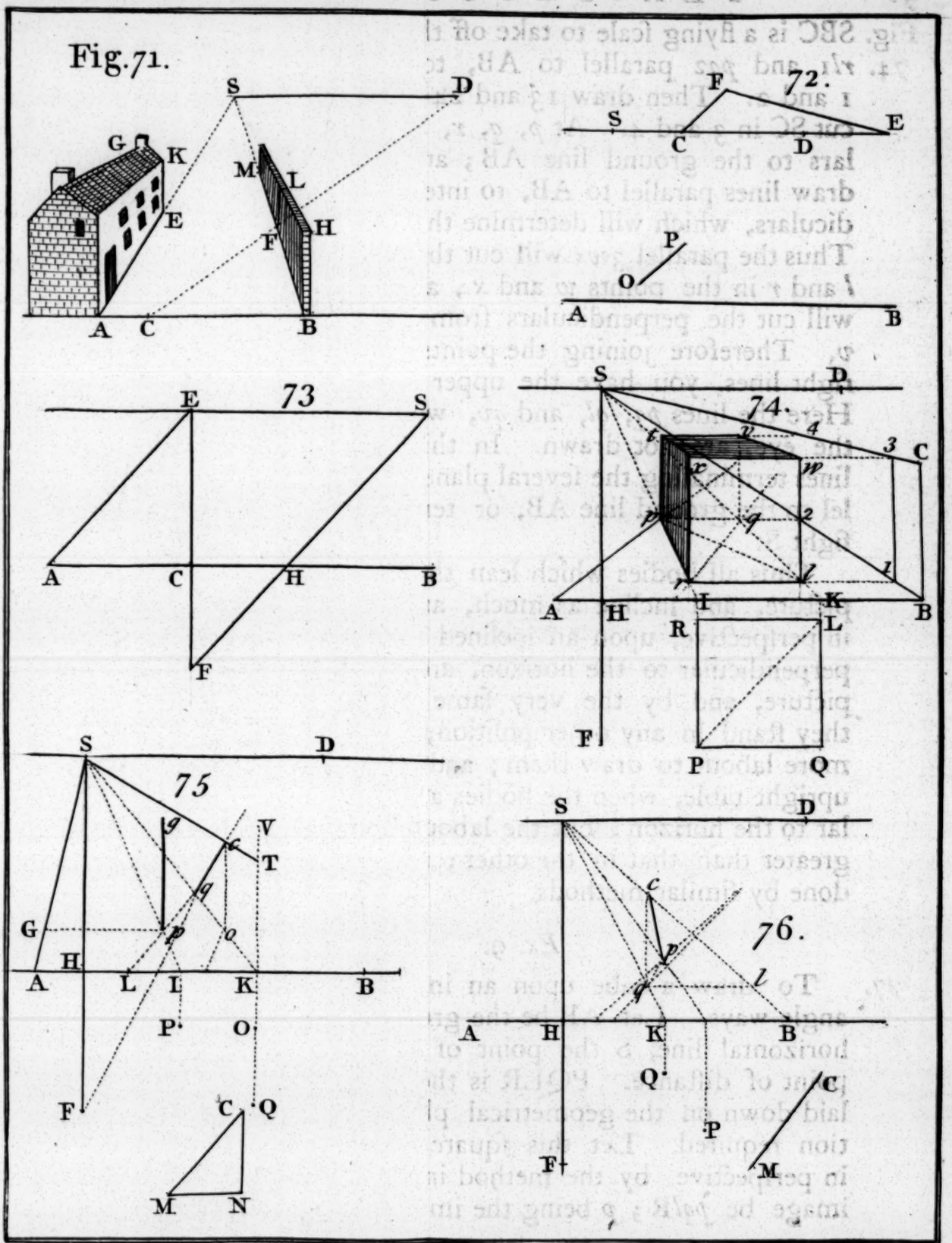
Fig. 74. SBC is a flying ſcale to take off the altitudes. Draw *rl*1 and *pq*2 parallel to AB, to interſect SB in 1 and 2. Then draw 13 and 24 parallel to BC, to cut SC in 3 and 4. At *p*, *q*, *r*, *l* raiſe perpendiculars to the ground line AB; and from 3 and 4 draw lines parallel to AB, to interſect theſe perpendiculars, which will determine the top of the ſolid. Thus the parallel 3*wx* will cut the perpendiculars at *l* and *r* in the points *w* and *x*; and the parallel 4*t* will cut the perpendiculars from *p* and *q* in *t* and *v*. Therefore joining the points *t*, *v*, *w*, *x* with right lines, you have the upper face of the ſolid. Here the lines *pq*, *ql*, and *qv*, which are hid from the eye, are not drawn. In this example all the lines terminating the ſeveral planes, are either parallel to the ground line AB, or tend to the point of ſight S.

Thus all bodies which lean the ſame way as the picture, and incline as much, are as eaſily drawn in perſpective, upon an inclined picture, as bodies perpendicular to the horizon, are upon an upright picture, and by the very ſame rules. But when they ſtand in any other poſition, there is required more labour to draw them; and ſo there is in an upright table, when the bodies are not perpendicular to the horizon; but the labour in one caſe is no greater than that in the other; and they are both done by ſimilar methods.

Ex. 9.

77. To draw a cube upon an inclined table, ſeen angle ways. Let AB be the ground line, SD the horizontal line, S the point of ſight, and D the point of diſtance. PQLR is the baſe of the cube laid down on the geometrical plane, in the ſituation required. Let this ſquare PQLR be drawn in perſpective, by the method in Ex. 4. and let its image be *pql*R; *p* being the image of P, *q* of Q,

and



Perspective. Pl. XIV. *pa. 96.*

the point R in the ground line Fig.
Then at the points P_1, V_1, K_1, we 77.
of the cube, which are perpen-
in the fold, but must pass thro'
point in the picture. Therefore
point I_1, of lines perpendicular
before. And from R draw lines
Then to terminate them at the
proceed by example 6, thus
perpendicular height of the cube,
ular to it, and draw CM, ma-
M equal to the angle which
with the horizon. Then take
, and let it perpendicular at
where S_1, SP_1, and S_2 in-
Let these perpendiculars
thus drawn from
the
the ground line,
points
point C_2
I_2, K_2
the
ing in the hy-
the vanishing line
being seen in C_2
made use of the
the same point
C_2

and *l* that of L; the point R in the ground line being common. Then at the points *p*, *q*, *l*, R, we must raiſe the ſides of the cube, which are perpendicular to the baſe in the ſolid, but muſt paſs thro' their accidental point in the picture. Therefore find the accidental point F, of lines perpendicular to the horizon as before. And from F draw lines thro' *p*, *q*, *l*, R. Then to terminate them at the top of the cube, proceed by examp. 6; thus. Let CN be the perpendicular height of the cube, draw MN perpendicular to it, and draw CM, making the angle CMN equal to the angle which the picture makes with the horizon. Then take CM in your compaſſes, and ſet it perpendicular at the points G, R, I, where S*p*, S*q*R, and S*l*, interſect the ground line. Let theſe perpendiculars reach to *g*, *r*, and *i*. Then thro' S and theſe points draw lines to interſect the lines drawn from F; that is, draw S*g* to interſect F*p* in the point 1, the top of the ſide *p*1. And draw S*r* to interſect F*q* and FR in 2 and 4; then 2 is the top of the ſide *q*2; and 4 is the top of the ſide R4. Alſo draw S*i* to interſect F*l* in 3, the top of the ſide *l*3. Then draw the lines 12, 23, 34, 41, for the upper face of the cube. Fig. 77.

One may obſerve, that ſince the ſides PR, RI, &c. make half a right angle with the ground line, the point of diſtance D, will be the accidental point of the ſides *pq*, R*l*, and alſo of their parallels 12, 43. And for the ſame reaſon the other point of diſtance will be the accidental point of R*p*, *lq*, 41, 32. And if the cube was in any other poſition, the accidental points might be eaſily found in the horizontal line SD; becauſe SD is the vaniſhing line of the baſe, and top of the cube, being both in or parallel to the geometrical plane.

In all theſe examples, I have made uſe of the ſame picture, in the ſame poſition, the ſame point

Fig. 77. of ſight and point of diſtance; and in general of the ſame profile of the picture as you have in fig. 73. And in all theſe examples the picture is ſuppoſed to lean backward; but all theſe things may as eaſily be done, if it leans towards you. In that caſe, the accidental point of lines perpendicular to the horizon will be above the point of ſight; but all operations will be the ſame. But for variety, I ſhall give one example where the picture leans forwards.

Ex. 10.

78. To draw a hexagonal right priſm, upon an inclined picture. We ſhall here ſuppoſe the picture to lean as much forward, as it before leaned backward, and that the height of the eye and the principal ray, remain the ſame. Let PQRTVZ be the hexagonal baſe deſcribed upon the geometrical plane, AB the ground line, SD the horizontal line, S the point of ſight, D the point of diſtance. Draw P1, Z2, V3 perpendicular to AB, the parallel ſides PQ, TV being ſo. From the point of ſight draw S1, S2, S3; by means of which, draw the baſe PQRTVZ in perſpective, which let be *pqrtvz*. Then to raiſe the images of the perpendiculars at the points *p*, *q*, *r*, *t*, *v*, *z*, proceed thus, at the points 1, 2, 3, raiſe the perpendiculars 16, 27, 38, each equal to the ſlant line CM, in the right-angled triangle MCN; where the angle CMN is the inclination of the picture, and CN the height of the priſm. Draw the line S8 from the point of ſight S; then from *v* and *t* draw lines towards F, (the accidental point of lines perpendicular to the horizon,) to cut S8 in the points 4 and 5, then *v*4, *t*5, repreſent two of the perpendicular ſides of the priſm. Again, draw the obſcure line S7; and from *z* and *r*, draw lines to F, to cut it in *o* and 9; then *zo*, *r*9 will be two more ſides of the priſm. Draw alſo the obſcure line S6; and from *p and q*, draw lines to F,

F, to cut S6 in the points *n* and *c*; then *pn*, *qc* are the two other ſides of the priſm; then joining all theſe points by right lines, you have the top of the priſm *nc*9540. And the planes being ſhaded, you have the true repreſentation of the priſm *vzpqn*5, upon the inclined picture. Fig. 78.

I ſuppoſe theſe examples are ſufficient to ſhew the method of drawing upon inclined pictures.

Scholium.

It appears from the practice, in all theſe examples, that the image of any original figure in the geometrical plane, will fall exactly in the ſame place of the picture, whether the picture lean forward or backward, or be upright; provided the principal ray, the length of the picture, and diſtance of the object from the ground line remains the ſame. But in objects elevated above the geometrical plane, the caſe is otherwiſe; and they will have a different appearance according to the different poſition of the picture. The principal uſe of drawing upon inclined pictures, is for painting any images upon the cieling of a room, or on any vaulted roof, or on the cupola of a church, &c. where the ſurface to be painted on, is in an oblique poſition.

As to the ſhadows of bodies; the rules are the ſame as in upright pictures; only inſtead of lines perpendicular to the horizon we muſt uſe lines parallel to the vertical line; and for lines perpendicular to the picture, make uſe of lines parallel to the line of ſtation or to the principal ray. In difficult caſes, the progreſs of the ſhadow may be traced out as it falls upon any planes, or upon any bodies, that are in its way; which is eaſily done from the ſituation of the light. And then theſe ſhadows, or parts of ſhadows, muſt be taken for

Fig. real objects, and ſo drawn in perſpective by the common rules.

PROB. XXVIII.

To draw the picture of any object mechanically.

This may conveniently be done by help of a candle or lamp. Let the object be placed near a wall, and a light, ſuch as a candle or lamp, placed at a good diſtance from it, or the ſun's light may be made uſe of; let this light project the ſhadow of the given object upon the wall behind it in lines nearly perpendicular to the wall. Then the ſhadow of the body would ſhew the appearance of the body to an eye placed where the light is, if the body was removed, and the ſhadow remained there. Therefore we muſt trace out all the out-lines of the ſhadow, with a pencil, upon the wall; which is very eaſy to do where they appear; but where the ſhadow of any part is intercepted by ſome other part; there will be ſome trouble. To find the ſhadow of any point, ſo placed as that its ſhadow cannot fall upon the wall; take a ſlender rod, and put one end of it to touch that point; and faſten the rod there by ſome means or other; then removing the body, whilſt the rod remains fixed; the ſhadow of the end of the rod will ſhew the place where the ſhadow of that point of the body would fall. Then replacing the body, you may proceed thus with any other point of the body, or as many points as you will. If any line in the object is hindered from caſting its ſhadow upon the wall; the place of the ſhadow will be found in like manner; lay a ſlender rod cloſe to the line, and mark the length with chalk; then remove the body, and where the ſhadow of that part of the rod, which you marked, falls upon the wall; there is the ſhadow of that

line

line in the object. And thus any lines in it may Fig.
be drawn.

Or the ſhadows of theſe points and lines may be found thus, take two ſtrings and extend them croſs one another; ſo that the ſhadow of their interſection may fall on the given point of the object. Then if the body be removed, the ſhadow of the interſection of the ſtrings, will fall upon that point of the wall where the ſhadow of the given point of the body would fall. In like manner extend a ſingle ſtring, ſo that a knot in it may fall upon the given point of the body, and fixing it there, remove the body, and the knot will caſt its ſhadow, where the ſhadow of the given point of the object would fall upon the wall. Alſo extend a ſtring, ſo that its ſhadow may fall upon any line in the body which we want to deſcribe; and mark the length on the ſtring, by tying two knots, or any other way. Then removing the body, that part of the ſtring will caſt a ſhadow upon the wall, where the ſhadow of the line in the object would fall. Other methods may be contrived, to get the ſhadows of ſuch points, as are hindered from falling on the wall. And ſome ſuch method may be contrived without removing the body, and being at the trouble to place it again. It may be thus, if your croſs ſtrings are long enough to caſt their ſhadows upon the wall, on every ſide of the body, whilſt their interſection caſts its ſhadow upon the given point of the object; continue theſe ſhadows behind the body, upon the wall, till they interſect; and it is evident, the point of interſection will be the place where the ſhadow of the given point of the body will fall. And thus you may find the ſhadows of as many points as you will, and conſequently of as many lines as you will, without ſtirring the body.

Inſtead of a light, the eye may be uſed, keeping it fixed in the place where the light ſhould be.

Fig. For the eye will project any point of the object upon the oppoſite wall which may be marked there by help of an aſſiſtant; and thus one part after another, and ſo the whole body, may be projected upon the wall. Alſo if one end of a ſtring be fixed where the light ſhould be, and taking hold of the other end, and drawing it ſtreight; if the ſtring be made to paſs along the out-lines of the object, the end of the ſtring, by help of a pencil, may be made to deſcribe the figure thereof upon the wall. And thus by means of the ſtring, the image of the body will be projected upon the wall, the ſame as by the ſhadow.

Otherwiſe.

79. To delineate any object; make a ſquare frame DE; divide the ſpace into ſmall ſquares, by threads fixed to the ſides, parallel to one another and to the ſides of the frame, and at equal diſtances. Set up a ſtand FH, upon the baſe FG; this muſt have a ſmall hole at H to look thro; it is fixed in the baſe, parallel to the frame DE. Alſo let the paper on which the draught is to be made, be divided into the like ſmall ſquares. Then placing the object beyond the frame in a proper ſituation; look at all the parts thereof ſucceſſively thro' the hole; and obſerve in what ſquares the ſeveral parts appear, and put them into the correſpondent ſquares in your paper; ſo at laſt you will have the picture of the whole object on your paper. Theſe ſquares may be drawn in black lead, and rubbed out afterwards.

There are alſo inſtruments contrived, by the help of which one may draw the appearance of any object; but the conſtruction of them is difficult, and therefore I ſhall ſay no more about them.

What is here ſaid of the projecting the image of a body upon a wall or any plane ſurface, is eaſily applicable

applicable to any curve ſurface. For wherever the ſhadow of any point falls by the light, there will be the image of that point. And likewiſe where the ſhadow of any line falls upon the curve ſurface, there will be the image of that line; for it will be where the plane paſſing thro' the luminous point and that line, cuts the curve ſurface. And the caſe is the ſame, if the figure is projected upon that curve ſurface, by rays drawn from the eye, placed where the light was. Fig.

PROB. XXIX.

To draw a deformed or monſtrous picture upon a plane, which ſhall appear regular, from a certain point.

Make a ſquare ABCD, and divide the ſides into as many equal parts as you will; and draw lines thro' all the points of diviſion, parallel to the ſides; theſe lines will divide the great ſquare into a number of little ſquares. In this ſquare draw the figure true which you want to appear deformed. 80. 81.

Again draw any line *ab*, which divide into as many equal parts as AB is divided into. From the middle point *e*, draw the line *e*V perpendicular to *ab*, and VS perpendicular to *e*V. Alſo draw the lines *a*V, *g*V, *h*V, *b*V, thro' the equal diviſions *g*, *h*, &c. Alſo draw *b*S, and where it cuts the former lines drawn to V, draw lines parallel to *ab*; then *abcd* is a deformed ſquare, and all the ſmall ſpaces repreſent the correſpondent little ſquares in the great one ABCD. Therefore in all the cells or little ſquares of *abcd*, draw the ſame part of the figure as you find in the correſpondent cells of ABCD; ſo you have the deformed image required. Then ſuppoſe VS to ſtand perpendicular to the plane *abcd*; if the figure be viewed by the eye

Fig. 80. 81. at S, it will appear in its true ſhape, as you have it in ABCD. But from any other point it will appear more or leſs deformed. And if the ſide *dc* was equal to the ſide DC, the figure would appear equally large, as that in ABCD would do, ſeen at the diſtance S*d*.

For if a ſquare whoſe ſide is *dc* be erected upon *dc*, and as many parallel lines drawn in it; and if SV be erected perpendicular to the plane *a*V*b*; then the firſt parallel line in the ſquare will be projected into the line 11, by the eye at S; and the ſecond parallel line into 22, the third into 33, and the laſt or top of the ſquare into *ab*; and the diagonal of the ſquare would be projected into *db*. Conſequently all the little ſquares would be projected into the correſpondent cells in the ſpace *abcd*. On the contrary, if the deformed figure *abcd* be given to be drawn in perſpective, its image will be the ſaid ſquare ſtanding upon *dc*, and all the little ſquares therein will be images of the reſpective cells in the figure *abcd*. And whatever is deſcribed in theſe cells, will appear in the little ſquares; and therefore the deformed picture drawn in the ſpace *abcd* will be a regular picture in the ſquare; that is, it will appear a regular head, ſeen from the point S in the air; but from any other point, the parts will appear to have different proportions, and therefore will be in ſome meaſure deformed.

This problem may be ſolved mechanically, by holding a candle at S in the air, and tracing out the ſhadow of the object (as the head here deſcribed) upon the plane ſurface given *abcd*, which may as eaſily be done upon a curve ſurface. But it is better if the candle ſhine thro' a ſmall hole, made in a plate ſet at S. The ſun's light may alſo be made uſe of for drawing either the original, or the deformed picture; which will be more or leſs ſo, accord-

according to the obliquity of the plane, it is to be drawn on. Fig. 80.

And by a like method, a deformed figure may be drawn upon the ſurface of a cone, or any ſolid; which will appear regular from a certain point of the axis. 81.

Cor. 1. *Hence, the trapezium abcd is the ſhadow of the perpendicular ſquare whoſe ſide is dc. And every particular cell is the ſhadow of the correſpondent ſmall ſquare in the upright one.*

For the ſhadow of any point of the ſquare ſtanding on *dc*, is where the line drawn from the luminous point S, cuts the plane V*ab*. And the ſhadow of any line in the ſquare, will be where the plane paſſing thro' that line and the luminous point, cuts the ſaid plane. Therefore the ſhadow of the top of the ſquare, is the line *ab*; the ſhadow of one ſide is *ad*, and that of the other ſide of the ſquare is *bc*; and the baſe *dc* coincides with the ſhadow. And thus the ſhadows of the lines or ſides of any ſmall ſquare, are the lines that circumſcribe the correſponding trapezium or cell. And conſequently the ſpace contained in each cell, is the ſhadow of the correſponding ſmall ſquare.

Cor. 2. *Hence, a picture that appears rude and irregular, when viewed directly, will appear regular and uniform, when viewed from a proper point.*

For the object *abcd* looked at directly, appears only a long trapezium; but to the eye of a ſpectator at S, it appears a regular ſquare. For the ſhape of the image and all the parts of it in the picture, to the eye; is the very ſame as if the rays had come at firſt from that picture ſtanding perpendicular upon *dc*; becauſe they come in the ſame lines whether they come from the picture, or from the original. And if the colours of the ſeveral parts, and the ſhadows be the ſame in both,

a perſon

Fig. 80. 81. a perſon will rather judge it to be regular than deformed; by being more accuſtomed to that, and ſeeming more natural.

Cor. 3. *The practice of making deformed objects from regular ones, is no more than the inverſe method of perſpective.*

For in the direct method of perſpective, the trapezium *abcd*, lying on the geometrical plane is projected into a ſquare upon the line *dc*, to the eye at S, in the air. So on the contrary, a ſquare in the picture is projected into the deformed image *abcd*, on the ground plane. In the firſt caſe the picture is between the eye and the object; but in the latter, they lie contrary.

Scholium.

Since ſhadows are no more than the deformations of objects falling upon ſome plane or other. Therefore the drawing of the ſhadows of bodies depends on the ſame rules, as finding their deformations. The rays of light paſſing by the outlines of a body project their ſhadows upon theſe planes; and the places where they fall will be found, by having the poſition of the light, and the ſhape of the body. Therefore the practice of theſe things depends upon ſome or other of the rules before laid down.

PROB. XXX.

To ſhew the uſes of perſpective in drawing.

The rules of perſpective before laid down, are employed either in repreſenting figures that lye in the geometrical plane, or objects that are elevated above that plane; and all objects whatever are either one or the other of theſe; and therefore the foregoing

foregoing rules are ſufficient for deſcribing any objects, and ought to be followed when we want any objects to be drawn perfectly exact, and mathematically true. And this is the beſt method when there are only right lined objects concerned, and but a few of them. But there are numerous objects that cannot be drawn this way, ſuch as mountains, trees, ſhips, &c. In ſuch caſes all we can do is to get a few of the principal points of ſuch objects, truly laid down; for to get all the points, is impoſſible, eſpecially where curve lines are concerned, and the figures are irregular; but the reſt of the work is to be done by hand. For the art of perſpective only lays down geometrical rules for finding the images of certain points, but as the number of points in any object is infinite, the work would be infinite; except in their ſingle caſe, where the points in the object happen to be connected by right lines; for then they will be right lines in the drawing, but in no other caſe; and therefore our only refuge is drawing by hand. Fig.

Then to gain a dexterity and exactneſs at drawing by hand; the beſt way is, firſt to draw ſome plain and ſimple objects in perſpective, and then try to imitate them by hand; and where you find any fault, rub it out, or elſe throw them away, and do them over again; ſtill endeavouring, to mend the former faults, and to make them more exact; which is as eaſy to do, as learning to write. When you have ſucceeded pretty well with one object, take a new one, and proceed the ſame way with that; and the like for as many as you pleaſe, till you improve in the art; and then you may make choice of more compounded objects which are harder to draw.

In objects that are more difficult to draw, let ſome of the principal lines be drawn by the rules of perſpective, and finiſh the reſt by hand; repeat this

Fig. this ſeveral times till you get them to your mind. Afterwards draw fewer of the lines in perſpective, and finiſh the reſt by hand. And laſt of all draw none of it in perſpective, but do the whole by hand; and that ſeveral times over, till it be tolerably exact. And thus by proceeding from eaſy objects to thoſe that are harder, you'll at laſt acquire a habit of drawing truly and readily. But it is practice alone that muſt compleat any perſon in this art of drawing, a multitude of rules ſignifying little or nothing.

When a complicated figure is to be drawn, it would be a difficult thing to do every ſmall part of it by the rules of perſpective; but by incloſing it in a ſquare divided into little ſquares, it may be done then by a perſon ſkilled in drawing. For where curve ſurfaces are concerned, the practical rules can only ſerve to find a few points thereof, or ſome right lines, that may incloſe it; by which means the drawer may then deſcribe them ſufficiently true.

From what has been ſaid, any one may ſee that it is in vain to endeavour, by the practical rules of perſpective, to deſcribe the hollows or riſings of objects, their lights and ſhadows, or any irregular turnings and windings thereof. Thus the capitals and ornaments of pillars, cannot be deſcribed that way, from the multitude of lines required to do it, and would never be exact. Nor can ſuch objects as have not any determinate ſhape, be deſcribed this way; ſuch as clouds, hills, trees, valleys, rivers, &c. but are far better done by hand. For in ſuch as theſe, there is more latitude allowed, and one can hardly miſtake ſo far as to make them appear monſtrous or unnatural. The turnings and windings of rivers, the ſhapes of trees, their branches and leaves; the limbs of animals, the features of men and women, the folds of their garments,

could

could never be performed by the ſtrict rules of perſpective. Fig.

We muſt be content therefore, to apply the Art of Perſpective to what it is capable of doing to perfection; and that is, laying down a general draught, which is to ſerve for a ground-work for our future operations; in which the ſeveral places and ſituations of objects are to be aſſigned. But then the finiſhing and beautifying the piece muſt be left to the ſkilful hand of the drawer. Indeed it is moſt uſeful where it is moſt wanted, and that is in deſcribing ſuch objects as are terminated by right lines, eſpecially perpendicular ones; ſuch as buildings, pieces of architecture, regular figures, and ſuch like; where a deviation from their true ſhape or poſition would be very ſenſible. In other caſes, all we can do by the mathematical rules, is to find a few points which ſerve for a guide to draw the reſt by. Thus in deſcribing a circle, we find as many points in it by rule and compaſs, as we think ſufficient, and then carry a curve thro' them by hand; ſo that ſtrictly ſpeaking no part of this image is found by geometrical rules, but theſe few points; all the reſt owes its being and exactneſs to the judgment and hand of the artiſt.

The principal uſe then of Perſpective, is for informing the judgment; for by being accuſtomed to draw objects in perſpective, one may better know which way ſuch and ſuch lines ſhould run, and where they ſhould end, and to what points they ſhould converge; and from thence judge better how an object ſhould appear, by ſhewing the relation between the object and its image; and by that means directs the hand to draw them, and the eye to diſcover any defect in the drawing, which otherwiſe would not be ſeen. Likewiſe when the general plan, and the principal parts are laid down exactly by rule; the minuter parts will naturally come

into

Fig. into their proper places, and all errors will then be eaſily avoided, and eaſily rectified. So that makeing uſe of the rules for drawing the principal parts, where they can conveniently be uſed; will give the picture ſuch a form as will guide the eye of the drawer, in deſcribing the other parts; and direct him to draw the reſt, and to finiſh the draught, and make it perfect and complete.

When objects are at a great diſtance, the magnitude muſt be diminiſhed in proportion; becauſe they appear under a leſſer angle than when nearer. And in ſhading them, the ſhadows muſt fall all one way, that is from the ſun. And in colouring them which ought to be done at laſt; the colours muſt be ſtronger near hand, and fainter further off; till at a great diſtance they loſe themſelves in a bluiſh miſt. And in all things you muſt perform the ſame thing in the picture, as you obſerve with your eye.

The buſineſs of *deſigning*, is the work of a maſter; this is forming a general idea of ſome large work, conſiſting of many parts, whoſe relation and connection, is entirely the invention of the artiſt. This may be ſome hiſtory piece, or contrived to ſhew ſome great tranſaction or event.

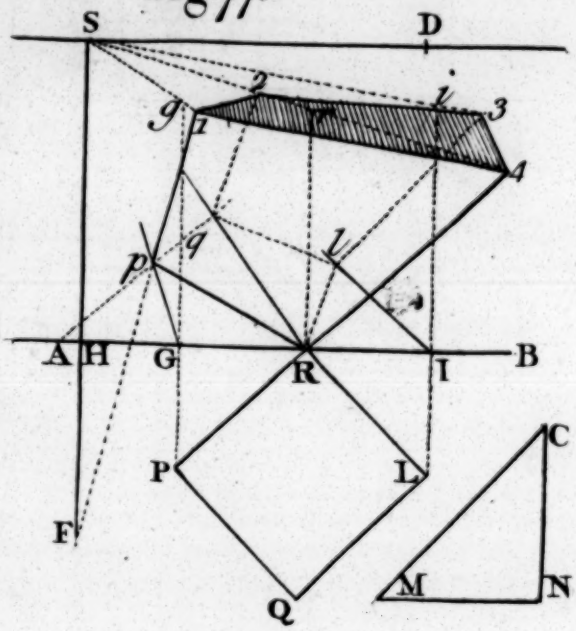
At firſt the artiſt muſt make only a rough draught of his deſign, and in a ſmall compaſs: this he is to correct and mend, as he ſees it neceſſary; and the outlines need only be drawn in black lead, to be rubbed out occaſionally. When he has fixed on the manner he would have it done, he may draw it larger and more compleat. And when he has got all the parts to his ſatisfaction, he muſt at laſt draw it, and all the parts of it, according to his deſign, and then proceed to the colouring it, if it is to be painted. Here drawing according to a deſign, is a different thing from drawing the pictures of objects, we have before us. For this picture

ture or deſign has no object but what exiſts in the ideas and mind of the artiſt. And therefore to make an elegant piece, the artiſt ſhould have a good invention, and be well acquainted with the nature of things, that he may proportion things truly, in his draught; and obſerve that the connection of the ſeveral parts be according to nature; and that they be pleaſing and inſtructive. To do this requires a good genius, and a great deal of practice. His deſign is formed entirely in his own head, which is to be executed by the rules beforementioned. And this branch of the art is gained by degrees, firſt by drawing pictures by ſight; and afterwards copying from good original draughts; obſerving to draw the outlines firſt, and then the inner parts, and then to ſhadow them. And laſt of all, they that would become complete, muſt make themſelves maſters of Perſpective. Fig.

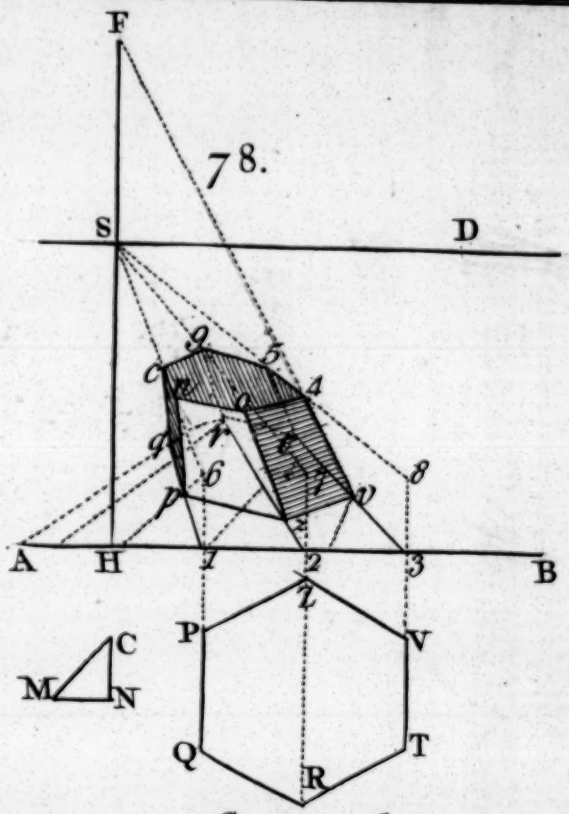
FINIS.

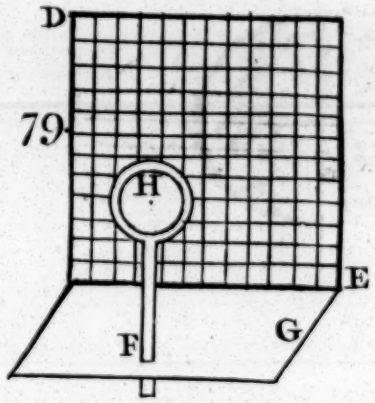

G.R.
III.
BRITISH MUSEUM

Fig. 77.

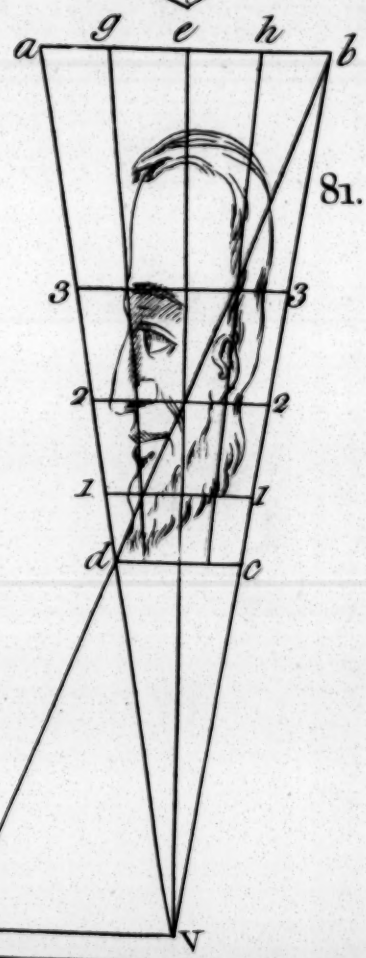


78.





80.



81.